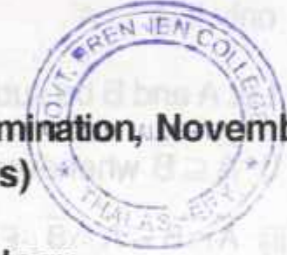




Reg. No. :

Name :

I Semester M.Sc. Degree (Reg./Sup./Imp.) Examination, November 2015
(2014 Admn. Onwards)
MATHEMATICS
MAT 1C04 : Basic Topology



Time : 3 Hours

Max. Marks : 60

PART - A

Answer **four** questions from this Part. **Each** question carries **3** marks.

1. Let X be an infinite set and let $T = \{U \in P(X) : U = \phi \text{ or } X - U \text{ is finite}\}$. Prove that T is a topology on X .
2. Is the usual topology on \mathbb{R} second countable? Justify your answer.
3. Define the subspace topology. If T is the usual topology on \mathbb{R} , describe the subspace topology on the set of all integers.
4. Let $X = \{1, 2, 3\}$, $T = \{\phi, \{1\}, \{1, 2\}, X\}$, $Y = \{4, 5\}$ and $U = \{\phi, \{4\}, Y\}$. Find a basis for the product topology on $X \times Y$.
5. Prove that the closed unit interval $[0, 1]$ has the fixed point property.
6. Determine whether the real line with the usual topology is compact. (4x3=12)

PART - B

Answer **four** questions from this Part without omitting any Unit. **Each** question carries **12** marks.

UNIT - I

7. a) Give an example of a set X and topologies T_1 and T_2 on X such that $T_1 \cup T_2$ is not a topology on X , justifying your claim.
- b) State and prove a necessary and sufficient condition for a family of subsets of a set X to be a basis for a topology on X .



8. a) Let A be a subset of a topological space (X, T) . Prove that A is closed if and only if $A' \subseteq A$.
- b) Let A and B be subsets of a topological space (X, T) . Prove that
- $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$ whenever $A \subseteq B$.
 - $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$. Further show by an example that the inclusion in part (ii) cannot be replaced by an equality.
9. a) If d is the usual metric on \mathbb{R} , prove that (\mathbb{R}, d) is complete.
- b) Prove that metrizable is a topological property.

UNIT – II

10. a) Prove that the property of being a Hausdorff space is hereditary, but being separable is not hereditary.
- b) Prove that every subspace of a separable metric space is separable.
11. a) Let (X_1, T_1) and (X_2, T_2) be topological spaces and let $(X_1 \times X_2, T)$ be the product space. Prove that the projections $\pi_1: X_1 \times X_2 \rightarrow X_1$ and $\pi_2: X_1 \times X_2 \rightarrow X_2$ are continuous. Further prove that the product topology is the smallest topology for which both projections are continuous.
- b) Define box topology and product topology on the product of an indexed family of topological spaces. For each $n \in \mathbb{N}$ let $X_n = \{1, 2\}$ and let T_n be the discrete topology on X_n . Let $X = \prod_{n \in \mathbb{N}} X_n$ and T be the product topology on X and U be the box topology on X . Show that $T \neq U$.
12. a) Let (X, T) be a topological space, let $\{(X_\alpha, T_\alpha): \alpha \in \Lambda\}$ be a collection of topological spaces and for each $\alpha \in \Lambda$, let $f_\alpha: X \rightarrow X_\alpha$ be a continuous function. Prove that the collection $\{f_\alpha^{-1}(U_\alpha): \alpha \in \Lambda \text{ and } U_\alpha \in T_\alpha\}$ is a basis for T if and only if $\{f_\alpha: \alpha \in \Lambda\}$ separates points from closed sets.
- b) Let $\{(X_\alpha, T_\alpha): \alpha \in \Lambda\}$ be an indexed family of first countable spaces, and let $X = \prod_{\alpha \in \Lambda} X_\alpha$. Prove that (X, T) is first countable if and only if T_α is the trivial topology for all but a countable number of α .



UNIT – III

13. a) Let T be the usual topology on \mathbb{R} . Prove that a subset of \mathbb{R} is connected if and only if it is an interval.
- b) Prove that the topological sine curve is not pathwise connected.
14. a) Let (X, T) be a connected space, let O be an open cover of X , and let a, b be distinct points of X . Prove that there is a simple chain consisting of members of O that connects a and b .
- b) Let (X, d) be a totally bounded complete metric space. Prove that (X, d) is compact.
15. a) Prove that every closed subset of a compact space is compact. Deduce that the Cantor set is compact.
- b) Prove that a topological space (X, T) is compact if and only if every family of closed subsets of X with the finite intersection property has a nonempty intersection. (4×12 = 48)