M 26474

Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.)
Examination, November 2014
MATHEMATICS
(2013 & earlier Admn.)
Paper – I : Algebra – I

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions. **Each** question carries **3** marks.

1. Find the order of (8, 4, 10) in the group $\mathbb{Z}_{12}\times\mathbb{Z}_{60}\times\mathbb{Z}_{24}$.
2. Find all Sylow 3-subgroups of S_4.
3. Give a presentation of $\mathbb{Z}_6$ involving one generator; involving two generators.
4. Show by an example that a field F′ of quotients of a proper subdomain D′ of an integral domain D may also be a field of quotients of D.
5. Factorize $x^4 + 4$ into a product of linear factors in $\mathbb{Z}_5[x]$.
6. Is $\mathbb{Q}[x]/\langle x^2 - 6x + 6\rangle$ a field ? Why ? **(4×3=12)**

PART – B

Answer **any four** questions without omitting **any** unit. **Each** question carries **12** marks.

Unit – I

7. a) Prove that the finite indecomposable abelian groups are exactly the cyclic groups with prime power order.

   b) If m divides the order of a finite abelian group, then prove that G has a subgroup of order m.

   c) Find all proper nontrivial subgroups of $\mathbb{Z}_2\times\mathbb{Z}_2$.

P.T.O.

8. a) Show that if H and N are subgroups of G, and N is normal in G, then $H \cap N$ is normal in G. Also prove that HN/N is isomorphic to $H/(H \cap N)$.

b) Let G be a finite group and let a prime p divides (G). Prove that G has a subgroup of order p.

9. a) State and prove the first Sylow theorem.

b) State (no proof) the third Sylow theorem. Show that no group of order 48 is simple.

Unit – II

10. a) If G is a nonzero free abelian group with a basis of r elements, prove that G is isomorphic to $\mathbb{Z} \times \mathbb{Z} \times ... \times \mathbb{Z}$ for r factors.

b) Let G be a nonzero free abelian group of finite rank n, and let K be a nonzero subgroup of G. Prove that K is free abelian of rank $s \leq n$.

11. a) Determine all nonabelian groups of order 8 upto isomorphism.

b) Let F be a field of quotients of an integral domain D and let L be any field containing D. Prove that there exists a map $\psi : F \rightarrow L$ that gives an isomorphism of F with a subfield of L such that $\psi(a) = a$ for $a \in D$.

12. a) Prove that the set R [x] of all polynomials in an indeterminate x with coefficients in a ring R is a ring under polynomial addition and multiplication.

b) Define the evaluation homomorphism ϕ_α for field theory and show that ϕ_α is indeed a homomorphism.

c) Let $\phi_\alpha : \mathbb{Z}_7[x] \rightarrow \mathbb{Z}_7$ be an evaluation homomorphism. Compute $\phi_3 (x^4 + 2x)(x^3 - 3x^2 + 3)$.

Unit – III

13. a) State (no proof) division algorithm for F [x], where F is a field. Show that (i) an element $a \in F$ is a zero of f(x) in F[x] if and only if $x - a$ is a factor of f(x) in F [x]. (ii) a nonzero polynomial f(x) in F [x] of degree n can have at most n zero in F.

b) Let $f(x) \in F[x]$, where F is a field and let f(x) be a polynomial of degree 2 or 3. Prove that f(x) is reducible over F if and only if it has a zero in F.

14. a) Let $\phi : R \rightarrow R'$ be a ring homomorphism with Kernel H. Then the additive cosets of H form a ring R/H whose binary operations are defined by choosing representatives : sum defined by $(a + H) + (b + H) = (a + b) + H$ and product defined by $(a + H)(b + H) = ab + H$. Also prove that the map $\mu : R/H \rightarrow \phi[R]$ defined by $\mu(a + H) = \phi(a)$ is an isomorphism.

b) Define an ideal of a ring R. Prove that the intersection of any two ideals of a ring R is an ideal of R. Is it true that union of any two ideals of R is again an ideal of R ? Justify.

15. a) Let R be a commutative ring with unity. Prove that M is a maximal ideal of R if and only if R/M is a field.

b) Define a principal ideal in a commutative ring with unity. Prove that every ideal in F[x], where F is a field is principal. Also describe the ideal $\langle x \rangle$ in F[x].

(4×12=48)

____