M 26475

Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com. Degree (Reg./Supp./Imp.) Examination, November 2014
(2013 & Earlier Admn.)
Mathematics
Paper – II : LINEAR ALGEBRA

Time : 3 Hours | Max. Marks : 60

Instructions : *1) Part – **A** : Answer **any four** questions. **Each** question carries 3 marks.*

*2) Part – **B** : Answer **any four** questions without omitting any Unit. **Each** question carries **12** marks.*

PART – A

1. Let T be the function from $\mathbb{R}^2$ into $\mathbb{R}^2$ defined by $T(x_1, x_2) = (x_2, x_1)$. Show that T is a linear transformation.

2. If M is an R-module and $x \in M$, then prove that the set $K = \{rx + nx : r \in R, n \in \mathbb{Z}\}$ is an R-submodule of M containing x.

3. In $\mathbb{R}^3$, let $\alpha_1 = (1, 0, 1)$, $\alpha_2 = (0, 1, -2)$, $\alpha_3 = (-1, -1, 0)$. If f is a linear functional on $\mathbb{R}^3$ such that $f(\alpha_1) = 1$, $f(\alpha_2) = -1$, $f(\alpha_3) = 3$, and if $\alpha = (a, b, c)$, find $f(\alpha)$.

4. Prove that similar matrices have the same characteristic polynomial.

5. State Cyclic Decomposition Theorem.

6. Let V be an inner product space, W a finite dimensional subspace, and E the orthogonal projection of V on W. Then prove that the mapping $\beta \rightarrow \beta - E\beta$ is the orthogonal projection of V on W^2.

P.T.O.

PART – B

UNIT – I

7. a) Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. If V is finite dimensional, then prove that rank (T) + nullity (T) = dim V.

b) Describe explicitly a linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^3$ which has its range the subspace spanned by (1, 0, –1) and (1, 2, 2).

8. a) Let T be a linear transformation from V into W. Then prove that T is non-singular if and only if T carries each linearly independent subset of V onto a linearly independent subset of W.

b) State and prove fundamental theorem of R-homomorphisms.

9. a) Let M be a free R-module with a basis $\{e_1, e_2, ..., e_n\}$. Then prove that $M \approx R^n$.

b) Let V be a non-zero finitely generated vector space over a field F. Then prove that V admits a finite basis.

UNIT – II

10. a) Let V be a finite dimensional vector space over the field F, and W be a subspace of V. Then prove that dim W + dim W^o = dim V.

b) If W_1 and W_2 are subspaces of a finite dimensional vector space, then prove that $W_1 = W_2$ if and only if $W_1^0 = W_2^0$.

11. a) Let T be a linear operator on an n-dimensional vector space V. Then prove that the characteristic and minimal polynomials for T have the same roots, except for multiplicities.

b) Find the minimal polynomial for the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$.

12. a) Let V be a finite dimensional vector space over the field F and Let T be a linear operator on V. Then prove that T is a triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.

b) Let W be an invariant subspace for T. Prove that the minimal polynomial for the restriction operator Tw divides the minimal polynomial for T, without referring to matrices.

UNIT – III

13. Let V be a finite dimensional vector space. Let $W_1, W_2, ..., W_k$ be subspaces of V and let $W = W_1 + W_2 + ... + W_k$. Then prove that the following are equivalent.

a) $W_1, W_2, ..., W_k$ are independent.

b) For each j, $2 \leq j \leq k$, we have

$w_j \cap (w_1 + w_2 + ... + w_{j-1}) = \{0\}$.

c) If $\mathscr{B}_i$ is an ordered basis for W_i, $1 \leq i \leq k$, then the sequence $\mathscr{B} = (\mathscr{B}_1, \mathscr{B}_2 .., \mathscr{B}_k)$ is an ordered basis for W.

14. State and prove primary decomposition theorem.

15. a) If V is an inner product space, then for any vectors α, β in V, prove that

i) $\|(\alpha|\beta)\| \leq \|\alpha\| \|\beta\|$ and

ii) $\|\alpha + \beta\| \leq \|\alpha\| + \|\beta\|$.

b) Prove that an orthogonal set of non-zero vectors is linearly independent.