Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com. Degree (Reg./Supple./Improv.) Examination, November 2014
MATHEMATICS (2013 & Earlier Admn.)
Paper IV : Topology – I

Time : 3 Hours   Max. Marks : 60

***Instructions** : 1) Answer **any four** questions from Part **A** . **Each** question carries **3** marks.*
*2) Answer **any four** questions from Part **B** ; without omitting **any** Unit. **Each** question carries **12** marks.*

PART – A

1. Determine the topology induced by a discrete metric on a set.
2. Let X be an indiscrete space. Verify whether every non-empty subset of X is dense in X.
3. Prove that every closed surjective map is a quotient map.
4. Show that the unit sphere

   $S^2 = \{x \varepsilon \mathbb{R}^3 : \|x\| = 1\}$ is path connected.
5. Is the real line $\mathbb{R}$ with usual topology compact ? Justify your answer.
6. Give an example of a space that is Hausdorff but not regular. (4×3=12)

PART – B

Unit – I

7. a) Define an open set in a metric space. In a metric space prove that
      i) The union of any family of open sets is open
      ii) The inter section of two open sets is open.

   b) Let X be a set and D a family of subsets of X. Prove that there is a unique topology τ on X such that it is the smallest topology on X containing D.

8. a) Define base for a topological space X. Show that the family of open sets $\mathscr{B}$ is a base for X if and only if for every x in X and every open set containing x, there exists $B \in \mathscr{B}$ such that $x \in B$ and $B \subseteq G$

   b) Let u denote that usual topology on $\mathbb{R}$. Let τ_1 denote the product topology on $\mathbb{R}^n$ and τ_2 the usual topology on it (induced by the Euclidean metric on $\mathbb{R}^n$). Prove that $\tau_1 = \tau_2$.

9. a) Define neighbourhood of a point x_0 in a topological space. Prove that a subset of a topological space is open if and only if it is a neighbourhood of each of its points.

   b) Let X, Y be topological spaces. Let $x_0 \in x$ and $f : X \to Y$ be a function. Prove that the following statements are equivalent (with usual notations)

      i) f is continuous at x_0

      ii) the inverse image under f of every neighbourhood of f (x_0) in Y is a neighbourhood of x_0 in X

      iii) for every subset $A \subseteq X, x_0 \in \overline{A} \Rightarrow f(x_0) \in \overline{f(A)}$

      iv) for every subset $A \subseteq X$, $x_0 \, \delta A \Rightarrow f(x_a) \, \delta f(A)$

Unit – II

10. a) Let $\{(Y_i, \tau_i); i \in I\}$ be an indexed family of topological spaces, X any set $\{f_i : i \in I\}$ be an indexed family of function from Y_i into X. Show that there exists a unique largest topology on X which makes each f_i continuous.

    b) Define a quotient map and quotient space of a topological space. Prove that the composite of two quotient maps is a quotient map.

    c) Prove that every quotient space of a discrete space is discrete.

11. a) Define a Lindeloff space. Let (X, τ) be a topological space and $A \subset X$. Prove that A is a Lindeloff subset of X if and only if the subspace $(A : \tau / A)$ is Lindeloff.

    b) Prove that every continuous real valued function on a compact space is bounded and attains its extrema.

12. a) Prove that a subset of $\mathbb{R}$ is connected if and only if it is an interval.

    b) Define a path-connected space and prove that every path-connected space is connected.

Unit – III

13. a) Prove that in a Hausdorff space, limits of sequences are unique.

    b) Prove that all metric spaces are T_4.

    c) Prove that every completely regular space is regular.

14. a) Prove that every regular Lindeloff space is normal.

    b) Suppose D is a decomposition of a space X each of whose members is compact and suppose that the projection $P : X \to D$ is closed. Prove that the quotient space is Hausdorff or regular according as X is Hausdorff or regular.

15. a) Let A, B be subsets of a space X and suppose there exists a continuous function $f : X \to [0,1]$, such that $f(x) = 0$ for all $x \in A$ and $f(x) = 1$ for all $x \in B$. Prove that there exists disjoint open sets U and V such that $A \subset U$ and $B \subset V$.

    b) State Tietze extension theorem.

    c) Let A be a closed subset of a normal space X and suppose $f : A \to (-1,1)$ is continuous. Prove that there exists a continuous function $F : X \to (-1, 1)$ such that $F(x) = f(x)$ for all $x \in A$ **(4x12=48)**

———————