M 26572

Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, November 2014
(2014 Admn. – under CBSS)
MATHEMATICS
MAT1C01 : Basic Abstract Algebra

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any 4** questions. **Each** question carries **3** marks.

1. Find a cyclic subgroup of maximum order in $\mathbb{Z}_{12} \times \mathbb{Z}_{16}$.
2. Prove that any group of order 2^n, where $n > 1$ has a nontrivial normal subgroup.
3. Verify the third isomorphism theorem for the subgroups $K = \langle 6 \rangle$ and $H = \langle 3 \rangle$ of $\mathbb{Z}_{18}$.
4. Find a subnormal series, with atleast five nontrivial subgroups, of the group of real numbers $\mathbb{R}$ under addition.
5. Find the Kernel of the evaluation homomorphism $\varphi_\alpha : \mathbb{Q}[x] \to \mathbb{R}$, when $\alpha = \pi$ and $\alpha = 3$
6. Find all zeros of the polynomial $x^4 + 2x^3 + 2x + 3$ in $\mathbb{Z}_4$.

PART – B

Answer **4** questions without omitting **any Unit**. **Each** question carries **12** marks.

Unit – I

7. a) Prove that the finite indecomposable abelian groups are exactly the cyclic groups with order a power of a prime. 6

   b) Consider an equilateral triangle T with vertices 1, 2 and 3; and sides S_1, S_2, and S_3. Describe the action of S_3 as the group of symmetrics, on T by describing the action of each element of S_3 on the vertices and sides of T. 6

P.T.O.

8. a) If X is a G – set and $x \in X$, prove that $|Gx| = (G : G_x)$. 8

b) Find all sylow 2-subgroups and sylow 3-subgroups of $\mathbb{Z}_{12}$. 4

9. a) Prove that if G is a finite group and p divides $|G|$, then the number of sylow p-subgroups of G is congruent to 1 modulo p and divides $|G|$. 9

b) Find all groups upto isomorphism of order p^2. 3

Unit – II

10. a) Prove that if D is an integral domain and F is a field containing D, then F contains a field of quotients of D. 9

b) Let K and N be normal subgroups of G with $K \vee N = G$, and $K \cap N = \{e\}$. Show that $G/K \simeq N$ and $G/N \simeq K$. 3

11. a) State the first isomorphism theorem. 2

b) Find isomorphic refinements of the series $\{0\} < \langle 5 \rangle < \mathbb{Z}_{40}$ and $\{0\} < \langle 4 \rangle < \mathbb{Z}_{40}$. 6

c) Prove that the group of symmetrics of the square D_4 is solvable. 4

12. a) Prove that a non-zero free abelian group of rank r is isomorphic to $\mathbb{Z} \times \mathbb{Z} \times ... \times \mathbb{Z}$ for r factors. 6

b) Is the group $\mathbb{Z}n$ free ? Explain. 3

c) Is $\{(2,0), (0, 2)\}$ a basis for $\mathbb{Z} \times \mathbb{Z}$? Why ? 3

Unit – III

13. a) Prove that a finite subgroup of the multiplicative group $\langle F^*, \cdot \rangle$ of a field F is cyclic. 8

b) Find a second degree polynomial in $\mathbb{Z}_4[x]$, which is a unit. 4

14. a) Prove that if $\varphi : R \to R'$ is a ring homomorphism, then ker (φ) is an ideal of R. 3

b) Describe all ring homomorphisms of $\mathbb{Z} \times \mathbb{Z}$ into $\mathbb{Z} \times \mathbb{Z}$. 6

c) Describe the ring $2\mathbb{Z}/4\mathbb{Z}$ by giving the addition and multiplication tables. Is if isomorphic to $\mathbb{Z}_2$? Why ? 3

15. a) Prove that if R is a commutative ring with unity, then an ideal M of R is a maximal ideal if and only if R/m is a field. 9

b) Find all $a \in \mathbb{Z}_3$ such that $\mathbb{Z}_3[x] / \langle x^2 + a \rangle$ is a field. 3

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