M 26574

Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, November 2014
(2014 Admission – Under CBSS)
MATHEMATICS
MAT 1C03 : Real Analysis

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions from this Part. **Each** question carries **3** marks.

1. Show that the union of R^n infinite collection of closed sets in a metric space need not be closed.

2. Let I = [0, 1] be the closed unit interval and f be a continuous mapping of I into I. Prove that f(x) = x for at least one $x \in I$.

3. Let f be defined for all real x and suppose that $|f(x) - f(y)| \leq (x - y)^2$ for all real x and y. Prove that f is constant.

4. If f(x) = 0 for all irrational x, f(x) = 1 for all rational x, then show that f is not Riemann integrable on [0, 1].

5. Let f be a function of bounded variation on [a, b]. Prove that f is bounded on [a, b].

6. Determine whether $f(x) = x \cos\left(\frac{\pi}{2x}\right)$ if $x \neq 0$, f(0) = 0 is of bounded variation on [0, 1].

P.T.O.

PART – B

Answer **any four** questions from this Part without omitting **any Unit**. **Each** question carries **12** marks.

UNIT – I

7. a) Define a metric space. For x, $y \in \mathbb{R}^1$, show that $d(x, y) = \frac{|x-y|}{1+|x-y|}$ defines a metric on $\mathbb{R}^1$.

b) Prove that every k-cell is $\mathbb{R}^k$ is compact.

8. a) Let E be a subset of $\mathbb{R}^k$. If every infinite subset of E has a limit point in E, then prove that E is closed and bounded.

b) If P is a nonempty perfect set in $\mathbb{R}^k$, then prove that P is uncountable. Deduce that the set of real numbers is uncountable.

9. a) Let f be a continuous mapping of a compact metric space X into a metric space Y. Prove that f is uniformly continuous on X.

b) Define discontinuities of the first and second kinds of a function at a point. Give an example of each type.

UNIT – II

10. a) State and prove the Taylor's theorem.

b) If $C_0 + \frac{C_1}{2} + ... + \frac{C_{n-1}}{n} + \frac{C_n}{n+1} = 0$, where $C_0, C_1, ..., C_n$ are real constants, prove that the equation $C_0, C_1 x + ... + C_n x^n = 0$ has at least one real root between 0 and 1.

11. a) Let f be a continuous mapping of [a, b] into $\mathbb{R}^k$ and f is differentiable in (a, b). Prove that there exists $x \in (a, b)$ such that $|f(b) - f(a)| \le (b - a) \mid f'(x) \mid$.

b) Prove that $f \in R(\alpha)$ on [a, b] is and only if for every $\varepsilon > 0$ there exists a partition P such that $U(P, f, \alpha) - L(p, f, \alpha) < \varepsilon$.

12. a) If f is continuous on [a, b], prove that $f \in R(\alpha)$ on [a, b].

b) Assume α increases monotonically and $\alpha' \in R[a,b]$. Let f be a bounded real function on [a, b]. Prove that $f \in R(\alpha)$ if and only if $f\alpha' \in R[a,b]$ and that $\int_a^b f\, d\alpha = \int_a^b f(x)\alpha'(x)\, dx$.

UNIT – III

13. a) Let f be Riemann integrable on [a, b]. For $a \le x \le b$, put $F(x) = \int_a^x f(t)\, dt$.

Prove that :

i) F is continuous on [a, b]

ii) If f is continuous at $x_0 \in [a, b]$, then F is differentiable at x_0 and $F'(x_0) = f(x_0)$

b) If f maps [a, b] into $\mathbb{R}^k$ and if $f \in R(\alpha)$ for some monotonically increasing function α on [a, b], prove that $|f| \in R(\alpha)$ and $\left|\int_a^b f\, d\alpha\right| \le \int_a^b |f|\, d\alpha$.

14. a) Let f be of bounded variation on [a, b] and $C \in (a,b)$ prove that f is of bounded variation on [a, c] and on [c, b] and that $V_f(a, b) = V_f(a, c) + V_f(c, b)$.

b) Let f be defined on [a, b]. Prove that f is of bounded variation on [a, b] if and only if f can be expressed on the difference of two increasing functions.

15. a) Define a path in $\mathbb{R}^n$. When it is said to be rectifiable ? With usual notations show that

$\Lambda_f(a,b) = \Lambda_f(a,c) + \Lambda_f(c,b)$ if $c \in (a,b)$.

b) Let $f : [a, b] \to \mathbb{R}^n$ be a path with components $f = (f_1, f_2, ..., f_n)$. Prove that f is rectifiable if and only if each components f_k is of bounded variation on [a, b].

c) Let f be defined by $f(t) = e^{2\pi it}$, $0 \le t \le 1$. Find the length of f.

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