M 26576

Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com. Degree (Regular/Supplementary/ Improvement) Examination, November 2014
(2014 Admission – under CBSS)
MATHEMATICS
MAT 1C05 : Differential Equations

Time : 3 Hours Max. Marks : 60

PART – A

Answer **four** questions from this Part. **Each** question carries **3** marks.

1. Show that the equation $4x^2y'' - 8x^2y' + (4x^2+1)y = 0$ has only one Frobenius series solution.

2. For the equation $y' = y$, find a power series solution of the form $\sum a_n x^n$.

3. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

4. Show that $x = e^{4t}$, $y = e^{4t}$ and $x = e^{-2t}$, $y = -e^{-2t}$ are two solutions of the system. $\frac{dx}{dt} = x + 3y$, $\frac{dy}{dt} = 3x + y$. Also find the particular solution of this system for which $x(0) = 5$ and $y(0) = 1$.

5. Find the critical points of the equation $\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{g}{a}\sin x = 0$ (The equation of motion for the damped vibrations of a pendulum of length a whose bob has mass m).

6. Does the initial value problem $y' = 3y^{2/3}$, $y(0) = 0$ has a unique solution in the interval $|x| < 1$? Explain why or why not.

P.T.O.

PART – B

Answer **any four** questions from this Part without omitting any unit. **Each** question carries **12** marks.

UNIT – I

7. a) Express $\sin^{-1} x$ in the form of a power series $\sum a_n x^n$ by solving $y' = (1-x^2)^{-1/2}$ in two ways.

   b) Find the general solution of

   $(1 - x^2)\ y'' - 2xy' + p(p+1)y = 0$ in terms of power series in x where p is a constant.

8. a) Show that the indicial equation of $x^2y'' + xy' + x^2y = 0$ has only root and deduce that $y = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2} x^{2n}$ is the corresponding Frobenius series solution.

   b) Find two independent Frobenius series solutions of $xy'' + 2y' + xy = 0$.

9. a) Define hypergeometric series and derive this series as a solution of Gauss's hypergeometric equation.

   b) Show that

   i) $F'(a, b, c, x) = \frac{ab}{c} F(a+1, b+1, c+1, x)$

   ii) $e^x = \lim_{b \to \infty} F\left(a, b, a, \frac{x}{b}\right)$.

UNIT – II

10. a) Establish the orthogonality property of the Legendre polynomials.

    b) Find the first three terms of the Legendre series of $f(x) = e^x$.

11. a) Show that $\frac{d}{dx}\left[x^P J_P(x)\right] = x\, J_{P-1}(x)$.

    b) Show that $J_m(x)$ and $J_{-m}(x)$ are linearly dependent when $m \geq 0$ is an integer.

    c) Assuming the formulas

    $\frac{d}{dx}\left[x^P J_P(x)\right] = x^P\, J_{P-1}(x)$ and $\frac{d}{dx}\left[x^{-P} J_P(x)\right] = -x^{-P}\, J_{P+1}(x)$, prove that

    $\frac{2P}{x} J_P(x) = J_{P-1}(x) + J_{P+1}(x)$. Also express $J_4(x)$ interms of $J_0(x)$ and $J_1(x)$.

12. a) If $x = x_1(t)$, $y = y_1(t)$ and $x = x_2(t)$, $y = y_2(t)$ are two linearly independent solutions on [a, b] of the system $\frac{dx}{dt} = a_1x + b_1y$, $\frac{dy}{dt} = a_2x + b_2y$, then prove that the general solution of the system on [a, b] is $x = c_1\, x_1(t) + c_2x_2(t)$, $y = c_1\, y_1(t) + c_2y_2(t)$ for any constants c_1 and c_2.

    b) Solve the system $\frac{dx}{dt} = x + y$, $\frac{dy}{dt} = 4x - 2y$.

UNIT – III

13. a) If the roots of the auxilary equation of the system $\frac{dx}{dt} = a_1x + b_1y$, $\frac{dy}{dt} = a_2x + b_2y$ where $a_1b_2 - a_2b_1 \neq 0$ are real, distinct and of the same sign, then prove that the critical point (0, 0) is a node.

    b) Determine the nature and stability properties of the critical point (0, 0) for the system $\frac{dx}{dt} = -4x - y$, $\frac{dy}{dt} = x - 2y$.

14. a) Show that (0, 0) is an asymptotically stable critical point for the system $\frac{dx}{dt} = -3x^3 - y$, $\frac{dy}{dt} = x^5 - 2y^3$.

    b) Find the exact solution of $y' = x + y$, $y(0) = 1$. Starting with $y_0(x) = 1$, apply Picardi method to calculate $y_1(x)$, $y_2(x)$, $y_3(x)$ and show that $y_n(x)$ converge to the exact solution.

15. a) Let f(x, y) be a continuous function that satisfies a Lipschitz condition $|f(x, y_1) - f(x, y_2)| \leq k\,|y_1 - y_2|$ on a strip $a \leq x \leq b$; $-\infty < y < \infty$. If (x_0, y_0) is any point of the strip, prove that $y' = f(x, y)$, $y(x_0) = y_0$ has exactly one solution $y = y(x)$ on the interval $a \leq x \leq b$.

    b) For what points (x_0, y_0) does the initial value problem $y' = |y|$, $y(x_0) = y_0$ has a unique solution on some interval $|x - x_0| \leq h$? Give reason.

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