Reg. No. : ....

Name : ....

M 8734

II Semester B.Sc. Degree (CCSS – Supple./Improv.) Examination, May 2015
(2013 and Earlier Admn.)
CORE COURSE IN MATHEMATICS
2B02 MAT : Foundations of Higher Mathematics

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) $2\left(x+\frac{x^3}{3}+\frac{x^5}{5}+....\right)=$ __________

   b) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...=$ __________

   c) The n-th term of the series $\frac{1}{1.3}+\frac{1}{2.5}+\frac{1}{3.7}+...$ is __________

   d) $\lim_{n\to\infty}\frac{1}{\left(1-\frac{1}{n}\right)^n}$. **(Weightage 1)**

2. Fill in the blanks :

   a) The dual of $(B\cap C)\cup A=(B\cup A)\cap(C\cup A)$ is __________

   b) If A = {1, 2, 3, 4}, B = {x, y, z} and if R is the relation from A to B defined by R = {(1, y), (1, z), (3, y), (4, x), (4, z)}, then $R^{-1}=$ __________

   c) If S = {a, b}, W = {1, 2, 3, 4, 5} and V = {3, 5, 7, 9} then $(S\times W)\cap(S\times V)$ is __________

   d) If the function $h:\mathbb{R}\to\mathbb{R}$ is defined by $h(x)=(x+3)$, which of the following ordered pairs does not belongs to graph of h : (4, 7), (–6, –9), (2, 6), (–1, 2) (4, 7), (–6, –9), (2, 6), (–1, 2). **(Weightage 1)**

P.T.O.

3. Sum the series $1 - \frac{1}{4} + \frac{1.3}{4.8} - \frac{1.3.5}{4.8.12} + ...$

4. If $n = \frac{1}{e} - \frac{1}{2e^2} + \frac{1}{3e^3}$ ...., show that $e^{n+1} - e - 1 = 0$.

5. Prove that $(A \cap B) \cup (A \cap B') = A$

6. If A = {a, b, c, d, e, f, g}, examine whether each of the following families of sets is a partition of A. Give reasons.

   a) $\{B_1 = \{a, c, e\}\}, \{B_2 = \{b\}\}, \{B_3 = \{d, g\}\}$

   b) $\{C_1 = \{a, e, g\}\}, \{C_2 = \{c, d\}\}, \{C_3 = \{b, e, f\}\}$

   c) $\{D_1 = \{a, b, e, g\}\}, \{D_2 = \{c\}\}, \{D_3 = \{d, f\}\}$

7. If '~' is a relation on the set of natural numbers defined by (a, b) ~ (c, d) if and only if a + d = b + c, then prove that '~' is an equivalence relation.

8. If R and R' are symmetric relations on a set A, prove that $R \cap R'$ is also a symmetric relation on A.

9. If the relation in N defined by "x divides y" is a partial order, then insert the correct symbol, <, > or || between each pair of numbers :

   a) 3 ....18 b) 18 24

   c) 9 ....3 d) 5 15

10. Define lattice. **(Weightage 5×1=5)**

Answer **any seven** from the following : **(Weightage 2 each)**

11. If R and R' are symmetric relations defined on a set A, prove that $R \cup R'$ is also a symmetric relation.

12. If f and g are functions defined on the set of all real numbers by $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$, find $f \circ g$ and $g \circ f$.

13. If $f : A \to B$ is an onto function and $g : B \to C$ is also onto, prove that $g \circ f$ is also onto.

14. If $f : A \to B$ and $g : B \to C$ have inverse functions $f^{-1} : B \to A$ and $g^{-1} : C \to B$, show that $g \circ f$ has an inverse function which is $f^{-1} \circ g^{-1} : C \to A$.

15. If $f : A \to B$ and $g : B \to C$. Prove the following :

   a) if $g \circ f$ is one-to-one, then f is one-to-one

   b) if $g \circ f$ is onto, then g is onto.

16. If f is a one-to-one and onto function defined on real numbers by $f(x) = 2x - 3$, find a formula that defines the inverse function f^{-1}.

17. If A = {2, 3, 4, ... } is order by "x divides y", then find (a) all minimal elements of A and (b) all maximal elements of A.

18. If L is a finite complemented distributive lattice, then show that every element a in L is a join of a unique set of atoms.

19. It α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of (i) $\sum \alpha^2\beta$ and (ii) $\sum \alpha^2$.

20. Find the equation whose roots are the roots of the equation $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ each diminished by 2. **(Weightage 7×2=14)**

Answer **any three** from the following : **(Weightage 3 each)**

21. If α, β, γ are the roots of $x^3 + px + q = 0$ form the equation whose roots are $\alpha^2 + \beta\gamma, \beta^2 + \gamma\alpha, \gamma^2 + \alpha\beta$.

22. Prove that $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!} = \frac{e}{2} + \frac{2}{e}$.

23. Show that $\frac{1}{2.3.4} + \frac{1}{4.5.6} + \frac{1}{6.7.8} + ... = \frac{3}{4} - \log 2$.

24. If $\alpha + \beta + \gamma = 3$, $\alpha^2 + \beta^2 + \gamma^2 = 5$, and $\alpha^3 + \beta^3 + \gamma^3 = 7$, form the cubic equation whose roots are α, β, γ.

25. Solve the equation $x^3 - 9x + 28 = 0$ by Cardan's method. **(Weightage 3×3=9)**
