K20U 1288

Reg. No. : ....

Name : ....

III Semester B.Sc. Degree (CBCSS – Sup.)
Examination, November 2020
(2014 – '16 Admns)
Core Course in Mathematics
3B03 MAT : ELEMENTS OF MATHEMATICS – I

Time : 3 Hours   Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. Write the negation of $\forall y \exists x (p(x) \wedge \sim q(y))$.
2. Give an example to show that $a^2 \equiv b^2$ (mod n) need not implies that $a \equiv b$(mod n).
3. State Newton's theorem on the sum of the powers of roots.
4. State fundamental theorem of algebra.

SECTION – B

Answer **any 8** questions from among the **5** to **14**. These questions carry **2** marks **each.**

5. Show that the set $\mathbb{Z}$ of all integers is denumerable.
6. Use De Morgan's Law, find the negation of Rahul likes apple and orange.
7. Find the polynomial equation of the lowest degree with rational coefficients having 3 and 1 – 2i as two of its roots.
8. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + ax^3 + bx^2 + cx + d = 0$, find the value of $\sum \alpha^2$.
9. Find the equation whose roots are two less than the roots of the equation $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$.

P.T.O.

10. Show that $x^{2n} = 10$ has $(2n - 2)$ complex roots.

11. Find the sum of $\cos(a) + \cos(a + b) + \cos(a + 2b) + \ldots + \cos(a + (n - 1)b)$.

12. Prove that if a/bc with gcd(a, b) = 1, then a/c.

13. Find the remainder when $1! + 2! + \ldots + 100!$ is divisible by 12.

14. If p is a prime and p/ab, then p/a or p/b.

SECTION – C

Answer **any 4** questions from among the **15** to **20**. These questions carry **4** marks **each**.

15. Suppose that S and T are sets and that $T \subseteq S$ then prove that
    i) Suppose S is a finite set, then T is a finite set.
    ii) If T is an infinite set, then S is an infinite set.

16. Show that the equation $x^3 + qx + r = 0$ has two equal roots if $27r^2 + 4q^3 = 0$.

17. Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

18. Solve $x^5 - x^4 - 4x^2 + 7x - 3 = 0$, given that it has multiple roots.

19. State and prove Cantor's theorem.

20. Show that cube of any integer is of the form 7k or $7k \pm 1$.

SECTION – D

Answer **any 2** questions from among the **21** to **24**. These questions carry **6** marks **each**.

21. Prove (i) There are infinitely many prime numbers. (ii) The product of any two odd integers is odd.

22. Prove that the sum of r^{th} powers of the roots of the equation $f(x) = 0$ is the coefficient of x^{-r} in the expansion of $\frac{xf'(x)}{f(x)}$ in descending powers of x.

23. Solve $x^3 + x^2 - 16x + 20 = 0$ by Cardan's method.

24. Using Euclidean algorithm, find gcd(12378, 3054) and express it as a linear combination of the two integers.