K20U 1289

Reg. No. : ....

Name : ....

III Semester B.Sc. Degree (CBCSS – Sup./Imp.) Examination, November 2020
(2017 – '18 Admns.)
CORE COURSE IN MATHEMATICS
3B03MAT – Elements of Mathematics – 1

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. If A is a set with m elements and B is a set with n elements and if $A \cap B = \varphi$ Then $A \cup B$ has ________ elements.
2. Give the remainder when f(x) is divided by x + a.
3. State fundamental theorem of algebra.
4. If a is an odd integer, the remainder when a^2 is divided by 8 is **(4×1=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Prove that the union of two disjoint denumerable set is denumerable.
6. Find the truth set T_p of the propositional function P(x), giving by "x + 2 > 7" on the set P{1, 2, 3, ...}.
7. Find a cubic equation with rational coefficients having the roots 1.3 + i2.
8. If α, β, γ are the roots of $2x^3 + 3x^2 - x - 1 = 0$. Find the equation whose roots are $\frac{1}{2\alpha}, \frac{1}{2\beta}, \frac{1}{2\gamma}$.
9. If $\alpha, \beta, \gamma, \delta$ are the roots of $ax^3 + 3bx^2 + 3cx + d = 0$. Find the value of $\Sigma(\beta - \lambda)(\gamma - \alpha)$.

P.T.O.

10. If p, q, r, s are positive show that $x^4 + qx^2 + rx - s = 0$ has one positive one negative and two imaginary roots.

11. Prove that a polynomial equation $f(x) = 0$ of degree n has exactly n roots.

12. If $\gcd(a, b) = 1$ prove that $\gcd(a + b, ab) = 1$.

13. Prove that there is an infinite number of primes.

14. Let $n > 1$ and a, b, c positive integers then Prove that (a) $a \equiv a \pmod{n}$ (b) $a \equiv b \pmod{n}$, $b \equiv c \pmod{n} \rightarrow a \equiv c \pmod{n}$. **(8×2=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**.

15. Prove that the Q of rational numbers is denumerable.

16. Solve $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$ given that the roots are in arithmetic progression.

17. Solve the reciprocal equation $2x^4 + x^3 - 17x^2 + x + 2 = 0$.

18. Solve the Diophantine equation $56x + 72y = 40$.

19. Find the remainder when $2^{20} + 3^{30} + 4^{40} + 5^{50} + 6^{60}$ is divided by 7.

20. Using the Sieve of Eratosthenes find all primes not exceeding 60. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. a) State and prove Cantor's theorem.

    b) Verify that the proposition $p \vee \neg(p \wedge q)$ is a taughtology.

22. If α, β, γ are the roots of $x^3 + qx + r = 0$. Find the equation whose roots are $\frac{\beta}{\gamma} + \frac{\gamma}{\beta}, \frac{\gamma}{\alpha} + \frac{\alpha}{\gamma}, \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

23. Solve $x^3 + 15x + 8 = 0$ using Carden's method.

24. If a and b are positive integers prove that g.c.d (a, b) 1.c.m(a, b) = ab.

(2×6=12)

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