



K17U 1968

Reg. No. :

Name :

Third Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination,
November 2017

CORE COURSE IN MATHEMATICS
(2014 Admn. Onwards)
3B03 MAT : Elements of Mathematics – I

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the first 4 questions are compulsory. They carry 1 mark each.

1. Give an example of a countable collection of finite sets whose union is not finite.
2. Find the sum of the cubes of the roots of the equation $x^4 + 2x + 3 = 0$.
3. State the fundamental theorem of algebra.
4. Give a prime number of the form $n^3 - 1$. (4×1=4)

SECTION – B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. If A_m is a countable set for each $m \in \mathbb{N}$, show that $\bigcup_{m=1}^{\infty} A_m$ is countable.
6. Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.
7. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, find the value of $\sum \alpha^3 \beta$.
8. Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in geometric progression.

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9. Solve the equation $x^3 - 6x^2 + 13x - 10 = 0$, given that the roots are in arithmetic progression.
10. Find the value of k for which $x^3 + 4x^2 + 5x + 2 + k = 0$ has equal roots.
11. Find the condition that all the roots of the equation, $x^3 + px + q = 0$ may be real.
12. Prove that $3a^2 - 1$ is never a perfect square.
13. Prove that the difference of two consecutive cubes is never divisible by 2.
14. Prove or disprove : Every positive integer can be written in the form $p + a^2$, where p is either a prime or 1 and $a \geq 0$. (8x2=16)

SECTION - C

Answer **any 4** questions from among the questions **15 to 20**. These questions carry **4 marks each**.

15. Let $C(x)$: x has a cat, $D(x)$: x has a dog and let $F(x)$: x has a ferret. Express the following statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers and logical operators. Let the domain consist of all students in your class.
 - a) A student in your class has a cat, a dog and a ferret.
 - b) All students in your class have a cat, a dog or a ferret.
 - c) Some student in your class has a cat and a ferret, but not a dog.
 - d) No student in your class has a cat, a dog and a ferret.
16. If the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ equals the sum of the other two, prove that $p^3 + 8r = 4pq$.
17. If α, β, γ be the roots of the equation $x^3 - 6x + 7 = 0$, form an equation whose roots are $\alpha^2 + 2\alpha + 3, \beta^2 + 2\beta + 3, \gamma^2 + 2\gamma + 3$.
18. Find the sum of the trigonometric series, $\sin \alpha + \frac{1}{2} \sin 2\alpha + \frac{1}{2^2} \sin 3\alpha + \dots$
19. Determine all solutions in the integers of the Diophantine equation, $56x + 72y = 40$.
20. Find the remainder when 41^{65} is divided by 7. (4x4=16)



SECTION - D

Answer **any 2** questions from among the questions **21 to 24**. These questions carry **6 marks each**.

21. Show that the following statements are equivalent :
 - a) S is a countable set.
 - b) There exists a surjection of \mathbb{N} onto S .
 - c) There exists an injection of S into \mathbb{N} .
22. Find $\frac{1}{\alpha^5} + \frac{1}{\beta^5} + \frac{1}{\gamma^5}$ where α, β, γ are the roots of the equation $x^3 + 2x^2 - 3x - 1 = 0$.
23. Solve : $x^4 - 3x^2 - 6x - 2 = 0$.
24. a) Show that the number $\sqrt{2}$ is irrational.
 b) Give an example to show that $a^2 \equiv b^2 \pmod{n}$ need not imply $a \equiv b \pmod{n}$. (2x6=12)