Reg. No. : ....

Name : ....

K19U 2473

III Semester B.Sc. Degree (CBCSS-Reg./Sup./Imp.) Examination, November - 2019
(2017 Admn. Onwards)
CORE COURSE IN MATHEMATICS
3B 03 MAT : ELEMENTS OF MATHEMATICS-I

Time : 3 Hours Max. Marks : 48

SECTION - A

All the first **4** questions are compulsory. They carry **1** mark each.

1. State True/False, the square of an odd integer is odd.
2. Find the remainder when $x^3 - 7x - 1$ is divided by $x + 2$.
3. State Sturm's theorem.
4. The greatest common divisor of -17 and 17 is?

SECTION-B

Answer any **8** questions from among the questions **5 to 14**. These questions carry **2** marks each.

5. If A_m is a countable set for each $m \in N$ prove that the union $A = \cup_{m=1}^{\infty} A_m$ is countable.
6. Prove that square of an odd integer is also an odd integer.
7. Form a polynomial equation of fourth degree with rational coefficients having one root $\sqrt{2} + \sqrt{-3}$.
8. If α, β, γ are the roots of $2x^3 + 3x^2 - x - 1 = 0$ find the equation whose roots are $\alpha\beta, \beta\gamma, \gamma\alpha$.
9. If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4 + px^3 + qx^2 + rx + s = 0$ find the value of $\sum(\alpha - \beta)^2$.
10. Show that $x^5 - 2x + 7 = 0$ has at least two imaginary roots.

P.T.O.

11. Find the sum of the trigonometric series $1-\frac{1}{2}\cos\alpha+\frac{1}{2}\frac{3}{4}\cos 3\alpha+........$

12. If a/c and b/c with gcd(a,b)=1 prove that ab/c

13. Prove that there is an infinite number of primes.

14. Find the remainder when 2^{50} is divided by 7.

SECTION-C

Answer any **4** questions from among the questions **15 to 20**. These questions carry **4** marks each.

15. State and prove Cantor's theorem.

16. Solve $x^4-8x^3+14x^2+8x-15=0$ given that the roots are in arithmetic progression.

17. Solve the reciprocal equation $x^4+6x^3-5x^2+6x+1=0$

18. Solve the Diophantine equation $172x+20y=1000$

19. Prove that the integer $53^{103}+103^{53}$ is divided by 39.

20. Find all prime numbers that divide 50!.

SECTION-D

Answer any **2** questions from among the questions **21 to 24**. These questions carry **6** marks each.

21. a) Prove that the Q of rational numbers is denumerable

    b) Show that the propositions $\neg(p\wedge q)$ and $\neg p\vee\neg q$ are logically equivalent.

22. If α, β, γ are the roots of $x^3+qx+r=0$ find the equation whose roots are $\frac{\beta}{\gamma}+\frac{\gamma}{\beta}, \frac{\gamma}{\alpha}+\frac{\alpha}{\gamma}, \frac{\alpha}{\beta}+\frac{\beta}{\alpha}$.

23. Solve $x^3+6x=20$ using carden's method.

24. Using Euclidean algorithm obtain the gcd(826, 1890) and find integers x and y such that gcd(826,1890) = 826x + 1890y .