M 7572

Reg. No. : ....

Name : ....

III Semester B.Sc. Degree (CCSS–Reg./Supple./Imp.) Examination, November 2014
COMPLEMENTARY COURSE IN MATHEMATICS
3C03 MAT : Differential Equations, Laplace Transforms, Fourier Series and Partial Differential Equations

Time : 3 Hours | Max. Weightage : 30

1. Fill in the blanks :
   a) Number of arbitrary constants in the solution of a first degree first order ordinary differential equation is ______________
   b) Laplace transform of coshat is ______________
   c) Period of cos x is ______________
   d) One dimensional heat equation is ______________ **(Weightage : 1)**

Answer **any six** from the following :

2. What do you mean by exact differential equation ?

3. Solve $\frac{dy}{dx} = \frac{x}{y}$.

4. Reduce the differential equation $y' + p(x)y = g(x)y^n$ to linear equation by using suitable substitution.

5. What do you mean by a self-orthogonal curve ?

6. Find Laplace transform of $\sin^3 2t$.

7. State second shifting theorem for Laplace transform.

P.T.O.

8. Find inverse Laplace transform of $\frac{s}{(s-1)(s-2)}$.

9. State half range Fourier Cosine series formula.

10. Verify that $u = e^{-t} \sin x$ satisfies one dimensional heat equation by assuming suitable value for the constant in the heat equation. **(Weightage : 6×1=6)**

Answer **any seven** from the following :

11. Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$.

12. Find the orthogonal trajectories of $xy = c$.

13. Using method of variation of parameters, solve $y'' + y = \tan x$.

14. Solve $\frac{dx}{dt} + 2x - 3y = 0$; $\frac{dx}{dt} - 3x + 2y = 0$.

15. Find the Laplace transform of $te^{-t} \cos t$.

16. Find inverse Laplace transform of $\frac{se^{-s/2} + \pi e^{-s}}{s^2 + \pi^2}$.

17. Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} \sin \omega t & 0 < t < \frac{\pi}{\omega} \\ 0 & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}, \quad f\left(t + \frac{2\pi}{\omega}\right) = f(t).$$

18. Find the Fourier sine series of $f(x) = x$ in $(0, 2)$.

19. Find a solution $u(x, y)$ of the partial differential equation $u_{xx} - u = 0$.

20. Using the method of separation of variables, solve the PDE $u_{xx} + u_{yy} = 0$.

(Weightage : 7×2=14)

Answer **any three** from the following :

21. Solve the initial value problem $y'' + 2y' + 5y = 1.25e^{0.5x} + 40\cos 4x - 55\sin 4x$, $y(0) = 0.2$, $y'(0) = 60.1$.

22. Using Laplace transform, solve $y''' + 2y'' - y' - 2y = 0$, $y(0) = 0$, $y'(0) = 0$ and $y''(0) = 6$.

23. Expand $f(x) = |x|$ in Fourier series in the interval $(-\pi, \pi)$. Also deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} - \ldots = \frac{\pi^2}{8}.$$

24. Find the Fourier series of period $2l$ for the function

$$f(x) = \begin{cases} l - x & 0 \le x \le l \\ 0 & l \le x \le < 2l \end{cases}.$$

25. Using the method of separation of variables, obtain the possible solution of one dimensional heat equation. **(Weightage : 3×3=9)**
