

K15U 0142

Reg. No. :

Name :

III Semester B.Sc./B.C.A. Degree (CCSS – Supple./Imp.)

Examination, November 2015

COMPLEMENTARY COURSE IN MATHEMATICS

3C03 MAT: Differential Equations, Laplace Transforms, Fourier Series and Partial Differential Equations

Time: 3 Hours Max. Weightage: 30

(2013 and Earlier Admission)

1. Fill in the blanks:

- a) Degree of the differential equation $\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = \sin x$ is ______
- b) Laplace transform of cos at is _____
- c) If f(-x) = f(x), the function is said to be _____
- d) Two dimensional Laplace equation is _____ (Weightage: 1

(Answer any six from the following):

- 2. Give an example for a Bernoulli's ordinary differential equation.
- 3. Solve $\frac{dy}{dx} = 1 + y^2$.
- 4. What do you mean by Wronskian of two functions y₁ and y₂?
- 5. Explain the procedure to find the orthogonal trajectory of a given curve.
- 6. State convolution theorem for Laplace transform.
- Find Laplace transform of cos²2t.
- 8. Find inverse Laplace transform of $\frac{1}{s^2 + 2s + 5}$.
- 9. State half range Fourier Sine series formula.
- 10. Verify that $u = x^2 + t^2$ satisfies one dimensional wave equation by assuming suitable value for the constant in the heat equation. (Weightage: $6 \times 1=6$)

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(Answer any seven from the following):

- 11. Solve (2x-4y+5)y'+x-2y+3=0.
- 12. Find the orthogonal trajectories of $y^2 = 4ax$.
- 13. Using method of variation of parameters, solve $y'' + 7y' 8y = e^{2x}$.
- 14. Solve $x^2y'' + xy' + y = \sin(\log x^2)$.
- 15. Find the Laplace transform of $\frac{1-e^t}{t}$.
- 16. Find inverse Laplace transform of $\frac{2s^2 6s + 5}{s^3 6s^2 + 11s 6}$
- 17. Find the Laplace transform of the periodic function $f(t) = \begin{cases} t & 0 < t < a \\ 2a t & a < t < 2a \end{cases}$ f(t + 2a) = f(t).
- 18. Find the Fourier cosine series of $f(x) = x^2$ in $(0, \pi)$.
- 19. Find a solution u(x, y) of the partial differential equation $u_{xy} + u_x = 0$.
- 20. Using the method of separation of variables, solve the PDE $u_{xx} u = 0$.

 (Weightage: $7 \times 2 = 14$)

(Answer any three from the following):

- 21. Solve the initial value problem $y'' + 2y' + y = e^{-x}$, y(0) = -1, y'(0) = 1.
- 22. Using Laplace transform, solve $y''' 3y'' + 3y' y = t^2e^t$, y(0) = 1, y'(0) = 0 and y''(0) = -2.
- 23. Find the Fourier series of $f(x) = x^2$, $-\pi < x < \pi$, given that f(x) is periodic with period 2π . Also deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$.
- 24. Expand f(x) = |x| in Fourier series in the interval (-l, l).
- Using the method of separation of variables, obtain the possible solution of one dimensional wave equation. (Weightage: 3x3=9)