



Reg. No. : .....

Name : .....



K16U 2251

**III Semester B.Sc. Degree (CCSS-Supple./Improve.)**  
**Examination, November 2016**  
**COMPLEMENTARY COURSE IN MATHEMATICS**  
**3C 03 MAT : Differential Equations, Laplace Transforms,**  
**Fourier Series and Partial Differential Equations**  
**(2013 and Earlier Admissions)**

Time : 3 Hours

Max. Weightage : 30

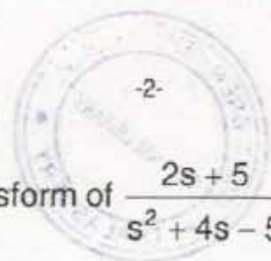
1. Fill in the blanks :

- a) Laplace transform of  $f(t) = 1$  is \_\_\_\_\_
- b) Laplace transform of  $f(t) = \sin 2t$  is \_\_\_\_\_
- c) Inverse Laplace transform of  $\frac{1}{s+a}$  is \_\_\_\_\_
- d) Inverse Laplace transform of  $\frac{1}{s^3}$  is \_\_\_\_\_ **(Weightage : 1)**

Answer **any six** from the following. (Weightage **1 each**).

- 2. What do you mean by a first order linear differential equation ? Give an example.
- 3. Solve the differential equation  $9y \frac{dy}{dx} + 4x = 0$ .
- 4. Solve  $(x^3 + 3xy^2) dx + (3x^2y + y^3) dy = 0$ .
- 5. Find Laplace transform of  $\cos^3 2t$ .

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6. Find inverse Laplace transform of  $\frac{2s+5}{s^2+4s-5}$ .
7. What do you mean by an even function? Give an example.
8. Write Euler's formula for Fourier series expansion of functions with period  $2\pi$ .
9. Show that  $u = \cos 4t \sin 2x$  is a solution to the one dimensional wave equation with suitable  $c$ .
10. Solve the partial differential equation  $u_{xx} - u = 0$ . (Weightage  $6 \times 1 = 6$ )

Answer any seven from the following. (Weightage 2 each).

11. Solve the initial value problem  $y'' + y' - 2y = 0$ ,  $y(0) = 4$ ,  $y'(0) = -5$ .
12. Solve  $x^3 y''' + 3x^2 y'' + x y' + y = 0$ .
13. Prove that the family of parabolas  $x^2 = 4a(y + a)$  is self orthogonal.
14. Define unit step function. Also find its Laplace transform.
15. Find the Laplace transform of periodic function of period  $T$ , defined by  $f(t) = \frac{kt}{T}$ ,  $0 < t < T$ .
16. Using convolution theorem, find inverse Laplace transform of  $\frac{s}{(s^2 + a^2)^2}$ .
17. Obtain the Fourier series expansion of  $f(x) = |x|$  in the interval  $(-\pi, \pi)$ .
18. Obtain the half range Fourier cosine series expansion of  $f(x) = x^2$  in  $0 < x < \pi$ .
19. Using separation of variables, solve  $u_{xx} + u_{yy} = 0$ .
20. Find the temperature in a laterally insulated bar of length  $L$  whose ends are kept at temperature 0, assuming that the initial temperature is

$$f(x) = \begin{cases} x & \text{if } 0 < x < \frac{L}{2} \\ L-x & \text{if } \frac{L}{2} < x < L \end{cases}$$

(Weightage  $7 \times 2 = 14$ )



Answer any three from the following. (Weightage 3 each).

21. Solve  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = \sin 3x + x^2$ .
22. Using method of variation of parameters, solve  $y'' + 4y = \sec 2x$ .
23. Using Laplace transform, solve the initial value problem  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t} \sin t$ ,  $y(0) = 0$ ,  $y'(0) = 1$ .
24. Obtain the Fourier series expansion of  $f(x) = \begin{cases} x & \text{if } 0 < x < \pi \\ \pi - x & \text{if } \pi < x < 2\pi \end{cases}$  and  $f(x + 2\pi) = f(x)$ .
25. Derive D'Alembert's solution to one dimensional wave equation. (Weightage  $3 \times 3 = 9$ )