K17U 2109

Reg. No. : ....

Name : ....

III Semester B.Sc. Degree (CCSS – Supplementary) Examination, November 2017
(2013 and Earlier Admissions)
COMPLEMENTARY COURSE IN MATHEMATICS
3C03 – MAT : Differential Equations, Laplace Transforms, Fourier Series and Partial Differential Equations

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) Laplace transform of $f(t) = t^2$ is __________

   b) Laplace transform of $f(t) = e^{-2t}$ is __________

   c) Laplace transform of $\frac{1}{s}$ is __________

   d) Laplace transform of $\frac{s}{s^2+4}$ is __________ **(Weightage : 1)**

Answer **any six** from the following. (Weightage **1 each**)

2. What do you mean by an exact differential equation ?

3. Solve the differential equation $\frac{dy}{dx} = 1 + y^2$.

4. Solve the differential equation $y' - y = e^{2x}$.

5. Find Laplace transform of sin 3t cos 2t.

6. Find inverse Laplace transform of $\frac{2s+5}{s^2+4s+13}$.

P.T.O.

7. What do you mean by a periodic function ? Give an example.

8. Write Fourier sine series expansion formula and Fourier cosine series expansion formula of functions in the interval (0, L).

9. Show that $u = e^{-4t}\cos 3x$ is a solution to the one dimensional heat equation with suitable c.

10. Solve the partial differential equation $u_{xy} + u_x = 0$. **(Weightage : 6×1=6)**

Answer **any seven** from the following. (Weightage **2 each**)

11. Solve the initial value problem $y'' - 4y' + 4y = 0$, $y(0) = 3$, $y'(0) = 1$.

12. Solve $y' + \frac{y}{3} = \frac{1}{3}(1-2x)y^4$.

13. Find the orthogonal trajectories of $x^2 + y^2 = 2ax$.

14. Find Laplace transform of :

$$f(t) = \begin{cases} 2 & \text{if} \quad 0 < t < \pi \\ 0 & \text{if} \quad \pi < t < 2\pi. \\ \sin t & \text{if} \quad t > 2\pi \end{cases}$$

15. Find inverse Laplace transform of $\log\left(\frac{s+1}{s-1}\right)$.

16. State and prove convolution theorem for Laplace transform.

17. Obtain the Fourier series expansion of $f(x) = x^2$, $f(x) = f(x + 2\pi)$ in the interval $(-\pi, \pi)$.

18. Obtain the half range Fourier sine series expansion of $f(x) = e^x$ in $0 < x < 1$.

19. Using separation of variables, solve $u_{xy} - u = 0$.

20. Find the solution of one dimensional wave equation corresponding to the triangular initial deflection.

$$f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2} < x < L \end{cases}$$ and initial velocity zero. **(Weightage : 7×2=14)**

Answer **any three** from the following. (Weightage **3 each**)

21. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \cos 2x + x^3$.

22. Solve the system of equations $\frac{dx}{dt} + 2y = \sin 2t; \frac{dy}{dt} - 2x = \cos 2t$.

23. Using Laplace transform, solve the initial value problem

$$\frac{d^2y}{dt^2} - y = t, \ y(0) = 1, \ y'(0) = 1.$$

24. Obtain the Fourier series expansion of $f(x) = x + x^2, -\pi < x < \pi$, $f(x + 2\pi) = f(x)$. Also deduce that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots = \frac{\pi^2}{6}.$$

25. Find the solution of one dimensional heat equation by Fourier series method.

(Weightage : 3×3=9)
