M 6329

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (CCSS – Regular/Supple./Improv.) Examination, May 2014

COMPLEMENTARY COURSE IN MATHEMATICS

4 C04 MAT : Numerical Analysis and Vector Calculus

Time : 3 Hours　　　　Max. Weightage : 30

1. Fill in the blanks :

   a) If the direction of $\bar{u}$ is constant, then $\bar{u} \times \frac{d\bar{u}}{dt} =$ __________

   b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then curl $\vec{r} =$ __________

   c) A vector $\vec{f}$ is said to be solenoidal if __________

   d) Formula for the flux of a three dimensional vector field $\vec{F}$ across an oriented surface S in the direction of $\hat{n}$ is __________ **(Weightage 1)**

Answer **any six** from the following (Weightage **1 each**) :

2. Using Newton-Raphson method, find the square root of 2.
3. What do you mean by interpolation ? State Newton's forward interpolation formula.
4. Apply Euler's method to solve the initial value problem $y' = x + y$, $y(0) = 0$ to find y (0.2) and y (0.4). Take $h = 0.2$.
5. Find by Taylor's series method the value of y at $x = 0.1$ from $\frac{dy}{dx} = x^2 y - 1$, $y(0) = 1$.
6. A particle is moving along a curve $x = e^t$, $y = 2 \cos 3t$, $z = 2 \sin 3t$, where t is the time variable ? Determine the velocity and acceleration at $t = 0$.
7. Find the gradient of $f(x, y) = y - x$ at the point (2, 1).

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8. Show that $\vec{f} = 2xye^z\hat{i} + x^2e^z\hat{j} + x^2ye^z\hat{k}$ is irrotational.

9. If $\vec{F} = 3xy\hat{i} - y^2\hat{j}$, evaluate $\int_C \vec{F}\cdot d\vec{r}$ where C is the curve in the xy-plane, $y = 2x^2$ from (0, 0) to (1, 2).

10. State Green's theorem in plane. **(Weightage 6×1=6)**

Answer **any seven** form the following. (Weightage **2 each**)

11. Using Gauss elimination method, solve the equations $x + 4y - z = -5; x + y - 6z = -12; 3x - y - z = 4$.

12. Using matrix inversion method, solve the equations $2x + y + z = 10; 3x + 2y + 3z = 18; x + 4y + 9z = 16$.

13. Using trapezoidal rule evaluate $\int_0^1 e^{-x^2}dx$ by dividing the interval into 10 sub-intervals.

14. Solve the differential equation $\frac{dy}{dx} = \log(x + y), y(0) = 2$ at x = 1.2 using Euler's modified method.

15. Using Picard's process of successive approximation, obtain the value of y at x = 0.4 from the equation $\frac{dy}{dx} = x^2 + y^2, y(0) = 0$.

16. If $\vec{A} = 5t^2\hat{i} + t\hat{j} + t^3\hat{k}$ and $\vec{B} = \sin t\,\hat{i} - \cos t\,\hat{j}$, find $\frac{d}{dt}(\vec{A}\cdot\vec{B}), \frac{d}{dt}(\vec{A}\times\vec{B})$ and $\frac{d}{dt}(\vec{A}\cdot\vec{A})$.

17. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, prove that $\nabla^2 r^n = n(n+1)\,r^{n-2}$.

18. If $\vec{a}$ is a constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\nabla\times(\vec{a}\times\vec{r}) = 2\vec{a}$.

19. Find the work done in moving a particle one round the circle $x^2 + y^2 = 9$ in the xy-plane if the field of force is $\vec{f} = (2x - y - z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$.

20. Evaluate $\iint_S \vec{f}\cdot d\vec{s}$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface bounding the region $x^2 + y^2 = 4$, z = 0 and z = 3. **(Weightage 7×2=14)**

Answer **any three** from the following (Weightage **3 each**).

21. Given that the values

x :	5	7	11	13	17
f(x) :	150	392	1452	2366	5202

Evaluate f(9) using Lagrange's interpolation formula.

22. Using Runge-Kutta method of fourth order, find an approximate value of y for x = 0.2 in steps of 0.1, if $\frac{dy}{dx} = x + y^2$, given that y = 1, where x = 0.

23. a) If $\vec{f}$ is a differential vector function and ϕ is a differential scalar function, then prove that $\nabla\cdot(\phi\vec{f}) = (\nabla\phi)\cdot\vec{f} + \phi(\nabla\cdot\vec{f})$.

b) If $\vec{F}$ is solenoidal, find curl curl curl $\vec{F}$.

24. A vector field is given by $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$. Show that the filed is irrotational and find its scalar potential.

25. Verify Stoke's theorem for the function $\vec{F} = (x^2 + y^2)\,\hat{i} - 2xy\,\hat{j}$ integrated round the rectangle in the plane z = 0 whose sides are along the lines x = a, x = −a, y = 0 and y = 6. **(Weightage 3×3=9)**