M 8564

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (CCSS-Reg./Supple./Imp.) Examination, May 2015
COMPLEMENTARY COURSE IN MATHEMATICS
4C04 MAT : Numerical Analysis and Vector Calculus

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) If the magnitude of $\bar{u}$ is constant, then $\bar{u} \cdot \frac{d\bar{u}}{dt} =$ ____________

   b) Tangent vector in the direction of $\bar{r}$ is ____________

   c) A vector point function $\bar{f}$ is said to be irrotational if ____________

   d) If $\bar{F}$ is a conservative field in a region D, then the value of $\int \bar{F} \cdot d\bar{r}$ around every closed loop in D is ____________ **(Weightage 1)**

Answer **any six** from the following : **(Weightage 1 each)**

2. Using Newton-Raphson method, find the positive solution of 2 sin x = x.

3. What do you mean by backward differences ? State Newton's backward interpolation formula.

4. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initial condition y = 1 at x = 0; find y for x = 0.02 and x = 0.04 by Euler's method.

5. Find by Taylor's series method the value of y at x = 0.2 for the differential equation $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$.

P.T.O.

6. A particle moves along the curve $\vec{r} = (t^3 - 4t)\,\hat{i} + (t^2 + 4t)\,\hat{j} + (8t^2 - 3t^3)\,\hat{k}$ where t denotes the time. Find the velocity and acceleration at $t = 2$.

7. Find the gradient of $f(x, y) = 2x + y^2 - 3$ at the point (1, 1).

8. Find the value of a so that the vector $\vec{f} = (x + 3y)\,\hat{i} + (y - 2z)\,\hat{j} + (x + az)\,\hat{k}$ is solenoidal.

9. Find the circulation of the field $\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$ around the circle $x^2 + y^2 = 1$, $z = 0$.

10. State Stoke's theorem. **(Weightage 6×1=6)**

Answer **any seven** from the following :**(Weightage 2 each)**

11. Using Gauss elimination method, solve the equations $2x + y + z = 10$; $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$.

12. Using matrix inversion method, solve the equations $3x + y + 2z = 3$; $2x - 3y - z = -3$; $x + 2y + z = 4$.

13. Using Simpson's rule evaluate $\int_1^2 \frac{dx}{x}$ by dividing the interval into 10 sub-intervals.

14. Solve the differential equation $\frac{dy}{dx} = x + \left|\sqrt{y}\right|$, $y(0) = 1$ at $x = 0.2$ using Euler's modified method.

15. Using Picard's process of successive approximation, obtain the value of y(0.1) from the equation $\frac{dy}{dx} = x - y^2$, $y(0) = 1$.

16. If $\vec{F}$ is a vector function of the scalar variable t, show that $\frac{d}{dt}\left[\vec{F}\,\vec{F}'\,\vec{F}''\right] = \left[\vec{F}\,\vec{F}'\,\vec{F}'''\right]$.

17. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, prove that $\nabla r^n = nr^{n-2}\,\vec{r}$.

18. If $u = x^2 + y^2 + z^2$ and $\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\text{div}\,(u\,\vec{v}) = 5u$.

19. Find the work done by $\vec{F} = (2y + 3)\,\hat{i} + xz\hat{j} + (yz - x)\,\hat{k}$ when it moves a particle from the point (0, 0, 0) to the point (2, 1, 1) along the curve $x = 2t^2$, $y = t$, $z = t^3$.

20. Apply Green's theorem to evaluate $\int_C \left[(2x^2 - y^2)\,dx + (x^2 + y^2)\,dy\right]$, where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$. **(Weightage 7×2=14)**

Answer **any three** from the following : **(Weightage 3 each)**

21. Given that the values

x :	5	7	11	13	17
f(x) :	150	392	1452	2366	5202

Evaluate f(9) using Newton's divided difference formula.

22. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = x + y$ with $y = 0$, where $x = 0$ at $x = 0.2$ and $x = 0.4$.

23. a) If $\vec{f}$ is a differential vector function and ϕ is a differential scalar function, then prove that $\nabla \times (\phi\vec{f}) = (\nabla\phi) \times \vec{f} + \phi\,(\nabla \times \vec{f})$.

b) If u is a scalar point functions, prove that $u\,\nabla u$ is irrotational.

24. A fluid motion is given by $\vec{V} = (y + z)\,\hat{i} + (z + x)\,\hat{j} + (x + y)\,\hat{k}$. Is the motion irrotational ? If so, find the velocity potential.

25. Verify divergence theorem for $\vec{F} = (x^2 - yz)\,\hat{i} + (y^2 - zx)\,\hat{j} + (z^2 - xy)\,\hat{k}$ over the rectangular parallelopiped, $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. **(Weightage 3×3=9)**