Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (CCSS-Supple./Imp.) Examination, May 2016
COMPLEMENTARY COURSE IN MATHEMATICS
4C04 MAT : Numerical Analysis and Vector Calculus
(2013 & Earlier Admissions)

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) If the vector function $\bar{u}(t)$ is constant, then $\frac{d\bar{u}}{dt}$ = ____________.

   b) If ϕ is a surface, then normal to the surface is ____________.

   c) A vector point function $\vec{f}$ is said to form a conservative field if ____________.

   d) If $\vec{f}$ is a conservative field and there exist a scalar function ϕ such that $\vec{f} = \nabla\phi$, then ϕ is known as ____________ of $\vec{f}$. **(Weightage 1)**

Answer **any six** from the following. **(Weightage 1 each)**

2. Using Newton-Raphson method, find a positive solution of $x^3 + x - 1 = 0$.

3. What do you mean by divided differences ? State Newton's divided difference interpolation formula.

4. Apply Euler's method to solve the initial value problem $y' = x + y$, $y(0) = 0$ to find $y(0.1)$ and $y(0.2)$. Take $h = 0.1$.

5. Solve $y' = y^2 + x$, $y(0) = 1$ using Taylor's series method and compute $y(0.1)$.

6. Find the angle between the tangents to the curve $x = 5t^2$, $y = t$, $z = 3 - t^3$ at the points $t = \pm 1$.

7. If $f(x, y, z) = x^2 + y^2 - 2z^2$, find ∇f at the point (1, 1, 1).

8. Find the values of the constants a, b, c so that

   $\vec{f} = (axy + bz^3)\hat{i} + (3x^2 - cz)\hat{j} + (3xz^2 - y)\hat{k}$ may be irrotational.

9. If $\vec{F} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, evaluate $\int_C \vec{F}.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t, y = t^2, z = t^3$.

10. State Divergence theorem. **(Weightage 6×1= 6)**

Answer **any seven** from the following. **(Weightage 2 each)**

11. Using Gauss elimination method, solve the equations $2x + 2y + z = 12$; $3x + 2y + 2z = 8$; $5x + 10y - 8z = 10$.

12. Using matrix inversion method, solve the equations $3x - y + z = 6$; $4x - y + 2z = 7$; $2x - y + z = 4$.

13. Using trapezoidal rule evaluate $\int_0^6 \frac{dx}{1+x^2}$ by dividing the interval into 6 sub-intervals.

14. Apply Euler's modified method to solve the initial value problem $y' = x + y$, $y(0) = 1$ to find $y(0.1)$.

15. Using Picard's method find approximate solution to the initial value problem $y' = 1 + y^2$, $y(0) = 0$.

16. If $\vec{F}$ is a vector function of the scalar variable t, show that $\frac{d}{dt}\left[\vec{F}\,\vec{F}'\vec{F}''\right] = \left[\vec{F}\,\vec{F}'\vec{F}'''\right]$.

17. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, prove that div (grad r^n) = $n(n + 1)\, r^{n-2}$.

18. If u and v are scalar point functions and $\vec{F}$ is a vector point function such that $u\vec{F} = \nabla v$, prove that $\vec{F}$. curl $\vec{F} = 0$.

19. Find the work done in moving a particle if the force field $\vec{f} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the curve defined by $x^2 = 4y$, $3x^3 = 8z$ from x = 0 to x = 2.

20. If C is a simple closed curve and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that $\int_C \vec{r}.d\vec{r} = 0$.

(Weightage 7×2= 14)

Answer **any three** from the following. **(Weightage 3 each)**

21. Given that the values

x :	20	25	30	35	40	45
f(x) :	354	332	291	260	231	204

Evaluate f (22) using Newton's forward interpolation formula.

22. Using Runge-Kutta method of fourth order, find an approximate value of y(0.1) and y(0.2) from $10\frac{dy}{dx} = x^2 + y^2$, given that y(0) = 1, taking h = 0.1.

23. a) If $\vec{f}$ and $\vec{g}$ are two differential vector functions, then prove that $\nabla.(\vec{f} \times \vec{g}) = \vec{g}.(\nabla \times \vec{f}) - \vec{f}.(\nabla \times \vec{g})$.

b) If $\vec{u}$ and $\vec{v}$ are irrotational, prove that $\vec{u} \times \vec{v}$ is solenoidal.

24. A vector field is given by $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$. Show that the field is irrotational and find its scalar potential.

25. Verify Green's theorem in the plane for $\int_C (xy + y^2)\,dx + x^2dy$ where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line y = x.

(Weightage 3×3=9)