**K19U 0723**

Reg. No. : ..............................

Name : ..................................

**IV Semester B.Sc. Degree (CCSS – Sup.) Examination, April 2019**
**(2013 and Earlier Admissions)**
**COMPLEMENTARY COURSE IN MATHEMATICS**
**4C 04 MAT : Numerical Analysis and Vector Calculus**

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) If $\vec{a} = (2x^2 - yz)\hat{i} + y^2z\hat{j} + (xy - 3z)\hat{k}$, then $\frac{\partial^2 \vec{a}}{\partial x \partial y} =$ ____________

   b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then div $\vec{r} =$ ____________

   c) A vector point function $\vec{f}$ is said to be irrotational if ____________

   d) If integral is independent of path joining A and B, then the value of

   $\int_A^B \vec{F} \cdot d\vec{r}$ is ____________ **(Weightage 1)**

Answer **any six** from the following (Weightage **1 each**) :

2. Using Newton-Raphson method, find the square root of 28 correct to three decimal places.

3. What do you mean by interpolation ? State Lagrange interpolation formula.

4. Explain Picard's method to solve first order differential equation $y' = f(x, y)$, $y(x_0) = y_0$.

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5. Apply Euler's method to solve the initial value problem $y' = x + y$, $y(0) = 0$ to find y(0.1) and y(0.2). Take h = 0.1.

6. A particle is moving along a curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5t^2$, where t is the time variable. Determine the velocity and acceleration at t = 1.

7. Find the normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1, 2, – 1).

8. Show that $\vec{f} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$ is irrotational.

9. Find the circulation of the field $\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$ around the circle $x^2 + y^2 = 1$, $z = 0$.

10. State Stoke's theorem. **(Weightage 6×1=6)**

Answer **any seven** from the following (Weightage **2 each**) :

11. Using Gauss elimination method, solve the equations $8y + 2z = -7$; $3x + 5y + 2z = 8$; $6x + 2y + 8z = 26$.

12. Using matrix inversion method, solve the equations $3x + y + 2z = 3$; $2x - 3y - z = -3$; $x + 2y + z = 4$.

13. Using Simpson's rule evaluate $\int_0^1 e^{-x^2}dx$ by dividing the interval into five sub-intervals.

14. Obtain the expressions for first and second order derivatives from Newton's forward difference interpolation formula.

15. Solve the differential equation $\frac{dy}{dx} = x + y$, $y(0) = 0$ at x = 0.2 using improved Euler method (take h = 0.2).

16. If $\frac{d\vec{a}}{dt} = \vec{w} \times \vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{w} \times \vec{b}$, prove that $\frac{d}{dt}(\vec{a} \times \vec{b}) = \vec{w} \times (\vec{a} \times \vec{b})$

17. Find the value of n, if $r^n\vec{r}$ is both solenoidal and irrotational where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$.



18. Prove that $\vec{f} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational and find its scalar potential.

19. If $\vec{f} = 3xy\hat{i} - y^2\hat{j}$, evaluate $\int_C \vec{f} \cdot d\vec{r}$ where C is the curve in the xy-plane $y = 2x^2$ from (0, 0) to (1, 2).

20. Apply Green's theorem to evaluate $\int_C [(\cos x \sin y - xy)dx + \sin x \cos y dy]$, where C is the circle $x^2 + y^2 = 1$. **(Weightage 7×2=14)**

Answer **any three** from the following (Weightage **3 each**) :

21. Find the value of y from the following data at x = 2.65.

| | | | | | |
|---|---|---|---|---|---|
| **x :** | – 1 | 0 | 1 | 2 | 3 |
| **y :** | – 21 | 6 | 15 | 12 | 3 |

22. Using Runge-Kutta method of fourth order, find an approximate value of y for x = 0.2 in steps of 0.1, if $\frac{dy}{dx} = xy + y^2$, given that y = 1, where x = 0.

23. a) If $\vec{f}$ is a differentiable vector function and $\phi$ is a differentiable scalar function, then prove that $\text{div}\left(\phi\vec{f}\right) = (\text{grad}\,\phi) \cdot \vec{f} + \phi(\text{div}\,\vec{f})$.

    b) If $\vec{u}$ and $\vec{v}$ are irrotational, prove that $\vec{u} \times \vec{v}$ is solenoidal.

24. Verify Stoke's theorem for the function $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface of sphere $x^2 + y^2 + z^2 = 1$.

25. Verify divergence theorem for $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$ over the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1. **(Weightage 3×3=9)**

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