K16U 0622

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (CBCSS – 2014 Admn. – Regular) Examination, May 2016

COMPLEMENTARY COURSE IN MATHEMATICS

4C04 MAT-PH : Mathematics for Physics and Electronics – IV

Time : 3 Hours | Max. Marks : 40

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. Find ∇f where $f(x, y) = \frac{x}{y}$

2. Evaluate $\int_C (dx + dy)$ where C is a smooth curve from point (1, 2) to (3, 4).

3. For the differential equation $y' = x + y^2$, find the first approximation to y given by Picard's method subject to the condition y = 1 when x = 0.

4. What is an initial value problem ? **(4×1=4)**

SECTION – B

Answer **any 7** questions from among the questions **5** to **13**. These questions carry **2 marks each.**

5. Find the tangent to the ellipse $\frac{1}{4}x^2 + y^2 = 1$ at $P : \left(\sqrt{2}, 1/\sqrt{2}\right)$.

6. Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at P : (2, 1, 3) in the direction of a = [1, 0, – 2].

P.T.O.

7. Find curl and divergence of the vector field $v = [e^x, e^{xy}, e^{xyz}]$.

8. Calculate $\int_C F(r) \cdot dr$ where $F = [x^2, y^2, 0]$, C the semicircle from (2, 0) to (– 2, 0), $y \geq 0$.

9. Use Green's theorem to evaluate $\int_C F(r) \cdot dr$ counterclockwise around the boundary curve C of the region R, where $F = [y \sin x, 2x \cos y]$, R the square with vertices $(0, 0), (\pi/2, 0), (\pi/2, \pi/2), (0, \pi/2)$.

10. Evaluate $\iint_S F \cdot n \, dA$ where $F = [x^2, 0, 3y^2]$ and S is the portion of the plane $x + y + z = 1$ in the first octant.

11. Find an approximate value of a real root of the equation $x^3 - x - 1 = 0$ by the bisection method.

12. Find a cubic polynomial which takes the following values : $y(1) = 24$, $y(3) = 120$, $y(5) = 336$, $y(7) = 720$.

13. Given $\frac{dy}{dx} = x^2 + y$; $y(0) = 1$, compute $y(0.02)$ using Euler's modified method. **(7×2=14)**

SECTION – C

Answer **any 4** questions from among the questions **14** to **19**. These questions carry **3** marks **each**.

14. Find the length of the circular helix $r(t) = [2 \cos t, 2 \sin t, 6t]$ from (2, 0, 0) to $(2, 0, 24\pi)$.

15. Evaluate $\iint_S F \cdot n \, dA$ by the divergence theorem where $F = [4x, 3z, 5y]$, S the surface of the cone $x^2 + y^2 \leq z^2, 0 \leq z \leq 2$.

16. Using Lagrange's interpolation formula, find the form of the function y(x) from the following table :

x	0	1	3	4
y	–12	0	12	24

17. Explain Simpson's 1/3-rule for numerical integration.

18. Use Newton-Raphson method to find a root of the equation $x^3 - 2x - 5 = 0$.

19. Given $y'' - xy' - y = 0$; $y(0) = 1$, $y'(0) = 0$, use Taylor's series method to determine y(0.1), correct to four decimal places. **(4×3=12)**

SECTION – D

Answer **any 2** questions from among the questions **20** to **23**. These questions carry **5** marks **each**.

20. Show that the helix [a cos t, a sin t, ct] can be represented by [a cos(s / K), a sin(s / K), cs/K] where $K = \sqrt{a^2 + c^2}$ and s is the arc length. Show that it has constant curvature $K = a / K^2$ and torsion $T = c / K^2$.

21. Verify Stokes's theorem for $F = \left[z^2, \frac{3}{2}x, 0\right]$, S the square $0 \leq x \leq a, 0 \leq y \leq a, z = 1$.

22. From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for x = 1.2.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

23. Given, $\frac{dy}{dx} = 1 + y^2$; $y(0) = 0$, use Runge-Kutta method with $h = 0.2$, to find y(0.2), y(0.4) and y(0.6). **(2×5=10)**