Reg. No. : ....

Name : ....

K16U 0623

IV Semester B.Sc. Degree (CBCSS – 2014 Admn.-Regular)
Examination, May 2016
COMPLEMENTARY COURSE IN MATHEMATICS
4C04MAT – CH : Mathematics for Chemistry – IV

Time : 3 Hours   Max. Marks : 40

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Show that the derivative of a vector function v(t) of constant length is either the zero vector or is perpendicular to v(t).

2. Show that the integral $\int_C F \cdot dr = \int_C (2x\,dx + 2y\,dy + 4z\,dz)$ is path independent in any domain in space.

3. Give the Newton's backward difference interpolation formula.

4. What do you mean by numerical integration ? **(4×1=4)**

SECTION – B

Answer **any 7** questions from among the questions **5** to **13**. These questions carry **2** marks **each**.

5. Find the tangent to the curve C : r(t) = [3 cos t, 3 sin t, 4t] at the point P : (3, 0, 8π).

6. Find the directional derivative of $f(x, y, z) = x^2 + y^2 + z^2$ at P : (2, –2, 1) in the direction of a = [–1, –1, 0].

7. If f(x, y, z) is a twice differentiable scalar function, show that div(grad f) = $\nabla^2 f$.

P.T.O.

8. Calculate $\int_C F(r) . dr$ where $F = [y^3, x^3]$, C the parabola $y = 5x^2$ from (0, 0) to (2, 20).

9. Let C be the positively-oriented boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$. Use Green's Theorem to evaluate

$$\int_C \left(y+e^{\sqrt{x}}\right) dx+\left(2x+\cos y^2\right)dy.$$

10. Obtain Green's theorem in the plane as a special case of Stoke's theorem.

11. Using Newton-Raphson method find a real root, correct to 3 decimal places, of the equation $\sin x = x/2$, given that the root lies between $\pi/2$ and π.

12. Given the differential equation, $y'=-y$ with the initial condition $y(0) = 1$, find $y(0.04)$, by Euler's method.

13. Solve the equation $y'=x+y^2$, subject to the condition $y = 1$ when $x = 0$, by Picard's method. **(7×2=14)**

SECTION – C

Answer **any 4** questions from among the questions **14** to **19**. These questions carry **3** marks **each**.

14. Determine the constants a and b such that $v = [2xy + 3yz, x^2 + axz - 4z^2, 3xy + 2byz]$ is irrotational.

15. Evaluate the line integral $\int_C F . r' ds$ by Stoke's theorem where C is the circle $x^2 + y^2 = 4, z = -3$, oriented counterclockwise and $F = [y, xz^3, -zy^3]$.

16. Using Newton's forward difference formula, find the sum $S_n = 1^3 + 2^3 + 3^3 + ... + n^3$.

17. Find a root of the equation $4e^{-x} \sin x - 1 = 0$ by Regula-Falsi method, given that the root lies between 0 and 0.5.

18. From the following table of values of x and y, obtain $\frac{dy}{dx}$ at $x = 0.6$.

x	0.4	0.5	0.6	0.7	0.8
y	1.5836	1.7974	2.0442	2.3275	2.6511

19. From the Taylor series for y(x), find y(0.1) correct to four decimal places if y(x) satisfies $y'=x-y^2$ and $y(0) = 1$. **(4×3=12)**

SECTION – D

Answer **any 2** questions from among the questions **20** to **23**. These questions carry **5** marks **each**.

20. Show that if C is represented by r(t) with arbitrary t, then the curvature is given

by $K(t) = \frac{\sqrt{(r'.r')(r''.r'')-(r'.r'')^2}}{(r'.r')^{3/2}}$.

21. Evaluate $\iint_S F . ndA$ where $F = [x, xy, z]$, S the complete boundary of $x^2 + y^2 \leq 1$, $0 \leq z \leq h$.

22. Evaluate $I= \int_0^{\pi/2} \sqrt{\sin x}\, dx$ using Simpson's $\frac{1}{3}$-Rule with $h = \pi/12$.

23. Solve the initial value problem defined by $\frac{dy}{dx}=\frac{3x+y}{x+2y}$, $y(1) = 1$ and find $y(1.2)$ and $y(1.4)$ by the Runge-Kutta fourth order formula. **(2×5=10)**