



Reg. No.:

Name:

**IV Semester B.Sc. Degree (CBCSS – 2014 Admn. Regular)
Examination, May 2016
Core Course in Mathematics
4B04 MAT : ELEMENTS OF MATHEMATICS – II**

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the first 4 questions are compulsory. They carry 1 mark each.

1. How many relations are there on the set {a, b, c} ?
2. Give an example of a finite partially ordered set having neither a first element nor a last element.
3. What do you mean by conjugate points with respect to a conic ?

4. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. (4×1 = 4)

SECTION – B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Give a relation which is neither symmetric nor antisymmetric and another one that is both symmetric and antisymmetric.
6. Let $A = \{1, 2, 3, \dots, 15\}$. Let R be the equivalence relation on A defined by congruence modulo 4. Find the equivalence classes determined by R .
7. Let $f, g : Z \rightarrow Z$ be functions defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. Obtain $f \circ g$ and $g \circ f$.



8. Show by example that a set which is not totally ordered may contain a linearly ordered subset.
9. Let $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be ordered by the relation "x divides y". Draw the Hasse diagram of A.
10. Find the coordinates of the point of intersection of tangents drawn to $y^2 = 4ax$ at the points where it is cut by the straight line $x \cos \alpha + y \sin \alpha = p$.
11. Find the condition for the lines $lx + my + n = 0$ and $l'x + m'y + n' = 0$ to be conjugate with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
12. Show that the product of the perpendiculars from any point of a hyperbola to its asymptotes is constant.
13. Find the value of x such that the matrix $A = \begin{bmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{bmatrix}$ has rank 2.

14. Reduce the matrix $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ to its normal form. (8x2 = 16)

SECTION - C

Answer **any 4** questions from among the questions **15 to 20**. These questions carry **4 marks each**.

15. Let $R = \{(a, b) \in P \times P : a \geq b \text{ and } a \leq 3\}$ where P is the set of positive integers. Is it a partial order? Justify your answer.
16. Let $f: R^+ \rightarrow [-5, \infty)$ be defined by $f(x) = 9x^2 + 6x - 5$ where R^+ is the set of positive real numbers. Find a formula for f^{-1} . Determine $(f^{-1} \circ f)(x)$ and $(f \circ f^{-1})(y)$.



17. Let $D = \{1, 2, 3, 5, 6, 10, 15, 30\}$ be ordered by divisibility.
- Which elements are join-irreducible?
 - Which elements are atoms?
 - Find a complement of 5, if it exists.
 - Express 30 as the join of a minimum number of irredundant join-irreducible elements.
18. Find the equation of the pair of tangents from (x_1, y_1) to the parabola $y^2 = 4ax$.
19. Chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touch $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$. Find the locus of their poles.
20. Using elementary row transformations, compute the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$$

(4x4 = 16)

SECTION - D

Answer **any 2** questions from among the questions **21 to 24**. These questions carry **6 marks each**.

21. Let A be a set of nonzero integers and let \approx be the relation on A defined by $(a, b) \approx (c, d)$ whenever $ad = bc$. Determine whether \approx is an equivalence relation on A. If it is so, find the equivalence class of $(3, 2)$.
22. Let L be a finite distributive lattice. Show that every $\alpha \in L$ can be written uniquely, except for order, as the join of irredundant join-irreducible elements.
23. Show that the locus of the mid-points of normal chords of the parabola $y^2 = 4ax$ is $\frac{y^2}{2a} + \frac{4a^3}{y^2} = x - 2a$.

24. For the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$, find non-singular matrices P and Q such

that PAQ is in the normal form.

(2x6 = 12)