K16U 0514

Reg. No. : .....

Name : .....

IV Semester B.Sc. Degree (CCSS – Supple./Imp.) Examination, May 2016
CORE COURSE IN MATHEMATICS
4B04 MAT : Calculus
(2013 and Earlier Admissions)

Time : 3 Hours Max. Weightage : 30

Fill in the blanks :

1. a) _____ is an example for a function which continuous at x = 0 and has no derivative at x = 0.

   b) $\frac{d}{dx}(1-x^2)^{-\frac{1}{2}} =$ _____

   c) If $\sqrt{5-2x^2} \le f(x) \le \sqrt{5-x^2}$ then $\lim_{x\to 0} f(x) =$ _____.

   d) The function $y = \sin\left(\frac{1}{x}\right)$ has no limit as $x \to$ _____. **W = 1**

2. a) $\int \frac{2z}{\sqrt[3]{z^2+1}}\,dz =$ _____

   b) $\int_{-1}^{1} 5x^4\sqrt{x^5+1}\,dx =$ _____

   c) $\Gamma(n) =$ _____ for $n \in N$.

   d) $\int_0^{\infty} e^{-x^2}dx =$ _____. **W = 1**

Answer **any five** from the following (Wt : **1** each).

3. Find :

   a) $\lim_{x\to 0} \frac{\sin x - x}{x^3}$ b) $\lim_{\theta\to 0} \frac{1}{\theta} - \frac{1}{\sin\theta}$

4. State maximum-minimum theorem for continuous function.

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5. Find two positive numbers whose sum is 20 and whose product is as large as possible.

6. State mean value theorem.

7. Show that the equation $x^3 + 3x + 1 = 0$ has exactly one root.

8. State Rolle's theorem.

9. Replace $(x-5)^2 + y^2 = 25$ by a polar equation.

10. Find b for which $f(x) = x^3 + bx^2 + cx + d$ have a point of inflexion at $x = 1$; where a, b, c, d are constants. **(5×1=5)**

Write **any 7** from the following (Wt : **2** each)

11. Find the n^{th} derivative of

    a) $e^x.\cos 2x$

    b) $\frac{x+1}{x^2-4}$

12. Find the asymptotes of $y^3 + x^2y + 2xy^2 - y + 1 = 0$.

13. Prove that the asymptotes of $x^2y^2 = c^2(x^2 + y^2)$ are the sides of a square.

14. Using Maclaurin's series, obtain the expansion of $e^x \sin x$ upto the term containing x^5.

15. Find the radius of curvature at (x, y) for the curve $a^2y = x^3 - a^3$.

16. Find the evolute of the parabola $y^2 = 4ax$.

17. Evaluate :

    a) $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2}$

    b) $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$

18. Find the length of the curve $y = \log \sec x$ between the points given by $x = 0$ and $x = \pi/3$.

19. One arch of the sine curve $y = \sin x$ revolves round the x – axis. Find the volume of the solid so generated.

20. Find the area enclosed by the cardioid $r = a(1 + \cos\theta)$. **(7×2=14)**

Write **any 3** from the following (Wt : **3** each).

21. Find $\frac{dy}{dx}$ for the following :

    a) If $x = a(\theta + \sin\theta)$; $y = a(1 - \cos\theta)$

    b) If $x^y = y^x$ prove that $\frac{dy}{dx} = \frac{y(y - x\log y)}{x(x - y\log x)}$.

    c) $y = (1+\log x)^{x^x}$.

22. State and prove fundamental theorem of calculus.

23. Use Simpson's rule with $n = 4$ to evaluate $\int_0^1 5x^4\,dx$.

24. Use reduction formula to evaluate

    a) $\int x^n e^{ax} dx$ b) $\int x^n \sin mx\,dx$

25. a) Find the perimeter of the cardioid $r = a(1 - \cos\theta)$

    b) Find the volume of the solid obtained by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line. **(3×3=9)**

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