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K19U 2276

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS- Reg./Sup./Imp.)

Examination, November-2019

(2014 Admn. Onwards)

Core Course in Statistics

5B 06 STA: MATHEMATICAL ANALYSIS - I

(Use of calculators and statistical tables are permitted)

Time : 3 hours | Max. Marks : 48

PART - A (Short Answer)

Answer **ALL the** questions. **(6×1=6)**

1. Define the limit superior and inferior of a sequence.

2. Discuss the monotonicity of the sequence $\{1/n\}$, $n \in N$.

3. Define an alternating series.

4. Discuss the continuity of the function $f(x)=\begin{cases} x+1 & \text{if } x<2 \\ 3 & \text{if } x \geq 2 \end{cases}$ at the point $x=2$.

5. Evaluate $\lim_{x \to 3} \frac{x^4-3^4}{x-3}$

6. Define differentiability of a function at a point.

P.T.O.

PART - B (Short Essay)

Answer any **SEVEN** questions. (7×2=14)

7. Show that for any real number x, $\lim_{n \to \infty} \frac{x^n}{n!} = 0$.
8. Show that every bounded sequence has a unique limit point.
9. Show that the series $\frac{1}{1^p} - \frac{1}{2^p} + \frac{1}{3^p} - \ldots\ldots$ converges for $p > 0$, using Leibnitz test.
10. Distinguish between absolute and conditional convergence.
11. Distinguish between continuity and uniform continuity.
12. Show that the function $f(x) = \begin{cases} x, when & x\ is\ irrational \\ -x, when & x\ is\ rational \end{cases}$ is continuous only at $x = 0$.
13. Discuss the continuity of the function $f(x) = [x]$ at the integer values of x.
14. State Darboux's theorem.
15. State Taylor's theorem.

PART - C (Essay)

Answer any **FOUR** questions. (4×4=16)

16. Show that the sequence $\{b_n\}$ where $b_n = \sum_{k=1}^{n} \frac{1}{\sqrt{n^2 + k}}$ converges to 1.
17. If $\{a_n\}$ and $\{b_n\}$ are two sequences with limits a and b respectively and $a_n \le b_n$, $\forall n$ then show that $a \le b$.
18. What do you mean by a positive term series? Explain with an example. Also state a necessary and sufficient condition for its convergence.
19. Test for the convergence of the series $\sum \frac{n^2 - 1}{n^2 + 1} x^n$ using, D' Alembert's Ratio test.
20. Prove or disprove: Every continuous function is uniformly continuous.
21. Establish the statement. Lagrange's mean value theorem is a particular case of Cauchy's mean value theorem.

PART - D (Long Essay)

Answer any **TWO** questions. (2×6=12)

22. State and prove Cauchy's general principle of convergence.
23. If $\sum u_n$ and $\sum v_n$ are two positive term series and $u_n \le kv_n, \forall n \ge m$, $k \ne 0$, then show that
    i) $\sum u_n$ is convergent if $\sum v_n$ is convergent
    ii) $\sum v_n$ is divergent if $\sum u_n$ is divergent
24. Show that every continuous function defined in a closed interval is necessarily bounded.
25. State and prove Rolle's Theorem. Examine the validity of the theorem for the function $f(x) = x^4 - 3x^2 + 1$ in $[-1,1]$.
