M 7152

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CCSS – Reg./Supple./Imp.) Examination, November 2014
CORE COURSE IN MATHEMATICS
5B09 MAT : Differential Equations and Numerical Analysis

Time : 3 Hours　　　　　　　　　　　　Max. Weightage : 30

1. Fill in the blanks :

   a) Characteristic equation of $y'' - y' + y = 0$ is ______________

   b) If $\lambda = 2$ and $\lambda = 3$ are the roots of the characteristic equation of $ay'' + by' + cy = 0$, then the general solution is ______________

   c) If Wronskian of $y_1(t)$ and $y_2(t)$ is zero, then $y_1(t)$ and $y_2(t)$ are ______________

   d) The equation $P(x)y'' + Q(x)y' + R(x)y = 0$ is exact if ______________

   (Weightage 1)

Answer **any six** from the following (Weightage **1 each**) :

2. Determine the order of the equation $\frac{d^2y}{dt^2} + \sin(t + y) = \sin t$. Also state whether the equation is linear or non-linear.

3. Solve $\frac{dp}{dt} = 0.5p - 150$.

4. Find the general solution of $y'' + y' + y = 0$.

5. Find the Wronskian of the vectors $x^{(1)}(t) = \begin{pmatrix} t \\ 1 \end{pmatrix}$ and $x^{(2)}(t) = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}$.

P.T.O.

6. Solve the boundary value problem $y'' + 2y = 0, y(0) = 1, y(\pi) = 0$.

7. Explain one dimensional wave equation.

8. Using Newton-Raphson method, find the square root of 2.

9. What do you mean by interpolation ? State Newton's forward interpolation formula.

10. Apply Euler's method to solve the initial value problem $y' = x + y, y(0) = 0$ to find $y(0.2)$ and $y(0.4)$. Take $h = 0.2$. **(Weightage : 6×1=6)**

Answer **any seven** from the following (Weightage **2 each**) :

11. Determine the value of r for which the differential equation $t^2y'' - 2ty' + 2y = 0$ has solution of the form $y = t^r, r > 0$.

12. Solve the initial value problem $ty' + 2y = 4t^2, y(1) = 2$.

13. Show that $y_1(t) = t^{\frac{1}{2}}$ and $y_2(t) = t^{-1}$ form a fundamental set of solution of $2t^2y'' + 3ty' - y = 0, t > 0$.

14. Find the particular integral of $y'' - 3y' - 4y = 3e^{2t}$.

15. Find the solution of the initial value problem $y'' + y' - 2y = 2t, y(0) = 0, y'(0) = 1$.

16. Using the method of separation of variables, solve one dimensional heat equation.

17. Find the solution u(x, y) of Laplace's equation in the rectangle $0 < x < a, 0 < y < b$ satisfying the boundary conditions $u(0, y) = 0, u(a, y) = 0, 0 < y < b; u(x, b) = 0, u(x, 0) = x(a - x), 0 < x < a$.

18. Using matrix inversion method, solve the equations $x + y + z = 6; 3x + y + z = 8; 2x + 2y - 3z = -7$.

19. Using trapezoidal rule evaluate $\int_0^1 e^{-x^2} dx$ by dividing the interval into 5 sub-intervals.

20. Using Picard's process of successive approximation, obtain a solution upto the fourth approximation from the equation $\frac{dy}{dx} = x + y, y(0) = 1$. **(Weightage : 7×2=14)**

Answer **any three** from the following (Weightage **3 each**) :

21. Solve the differential equation $(y \cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0$.

22. Solve the initial value problem $y' = 2t(1 + y), y(0) = 0$ by the method of successive approximation.

23. Using method of variation of parameters, solve $y'' + 4y = \tan 2t$.

24. Given that the values

x :	5	7	11	13	17
f(x) :	150	392	1452	2366	5202

Evaluate f(9) using Lagrange's interpolation formula.

25. Using Runge-Kutta method of fourth order, find approximate values of y(0.1) and y(0.2) from $\frac{dy}{dx} = x + y^2$, given that $y(0) = 1$. **(Weightage : 3×3=9)**
