



K20U 0128



Reg. No. :

Name :

VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.)
Examination, April 2020
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B11MAT : Numerical Methods and Partial Differential
Equations

Time : 3 Hours

Max. Marks : 48

SECTION - A

All the 4 questions are compulsory. They carry 1 mark each.

1. State the intermediate value theorem for finding the real root of an equation.
2. Complete the expression $\Delta = E -$
3. Give the maximum bound for error $R_1(f)$ in trapezoidal rule.
4. For a function $u(r, \theta, t)$, give its Laplacian in Polar co-ordinates.

SECTION - B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Find $\sqrt{15}$ by Bisection method, correct to two decimal places.
6. Find a root of the equation $\log x - \cos x = 0$, where x is in radians, correct to two decimal places, using Regula Falsi method.
7. Show that $\Delta \left(\frac{f}{g} \right) = - \frac{g_1 \Delta f - f_1 \Delta g}{g g_{1+1}}$

P.T.O.



8. Find $\log_e(2.7)$ from the following table using Lagrange's interpolation formula.

| | | | |
|-------------------------------|--------|--------|--------|
| x | 2 | 2.5 | 3 |
| $\log_e(x)$ | 0.6932 | 0.9163 | 1.0986 |

9. Evaluate $\sqrt{\frac{2}{\pi}} \int_0^1 e^{-x^2/2} dx$, using Simpson's 1/3 rule, taking $h = 0.25$.
10. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Trapezoidal rule with $h = 0.25$.
11. Find a solution to the initial value problem $y' = 2y - x$, $y(0) = 1$, by performing two iterations of the Picard's method.
12. Find $y(1.2)$, given the differential equation $y' = -2xy^2$, with the condition $y(1) = 1$, using Taylor's series with step size $h = 0.1$.
13. Give the Fourier series solution of the one dimensional wave equation, with fixed ends and initial conditions $u(x, 0) = f(x)$ and $\frac{\partial u}{\partial t} \Big|_{t=0} = g(x)$.
14. Solve the equation $u_{yy} = 0$ where u is a function of x and y .

SECTION - C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Find a real root of the equation $x^3 + x^2 - 1 = 0$ by General iteration method, correct to two decimal places.
16. Using Newton's divided difference formula, find a cubic polynomial for the following data. Hence find $f(3)$.

| | | | | |
|-------------|---|---|---|---|
| x | 0 | 1 | 2 | 4 |
| f(x) | 1 | 1 | 2 | 5 |

17. The function $f(x)$ represented by the following data has a minimum in the interval $(0.5, 0.8)$. Find this point of minimum and the minimum value.

| | | | | |
|-------------|--------|--------|--------|--------|
| x | 0.5 | 0.6 | 0.7 | 0.8 |
| f(x) | 1.3254 | 1.1532 | 0.9432 | 1.0514 |



18. Find the approximate value of $y(0.1)$ given that $y' = x^2 + y^2$, $y(0) = 1$ using three iterations of the Modified Euler's method with $h = 0.1$.
19. Given $\frac{dy}{dx} = y - x$ with $y(0) = 2$, use Runge - Kutta method of order two to find $y(0.2)$ taking $h = 0.1$.
20. A stretched string of length l and fixed end points has initial displacement $y = a \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Find the vertical displacement $y(x, t)$ at any distance x from one end at time t .

SECTION - D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Find an interval of unit length which contains the smallest positive root of the equation $e^x - 2x^2 = 0$. Hence find the root of this equation using Newton - Raphson method correct to three decimal places.
22. The following table gives the value of e^{-x} for some values of x :

| | | | | | | | |
|----------------------------|--------|--------|--------|--------|--------|--------|--------|
| x | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |
| e^{-x} | 0.8187 | 0.7408 | 0.6703 | 0.6065 | 0.5488 | 0.4966 | 0.4493 |

Determine the value of $e^{-0.55}$ using Stirling's central difference formula.

23. Compute $f'(0.2)$ and $f''(0)$ from the following table.

| | | | | | | |
|-------------|------|------|------|-------|-------|--------|
| x | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
| f(x) | 1.00 | 1.16 | 3.56 | 13.96 | 41.96 | 101.00 |

24. Find the temperature $u(x, t)$ in a slab of length L whose ends are kept at zero temperature and whose initial temperature $f(x)$ is given by

$$f(x) = \begin{cases} k, & \text{when } 0 < x < \frac{L}{2} \\ 0, & \text{when } \frac{L}{2} < x < L \end{cases}$$