G-NOLIDER

9. State and prove the Fundamental Theorem of Calculus (First Form).

2. Let I = [a,b] and Mt c = (a,b) Let  $I = I \to R$  be a bounded function. Then prove that I is integrable on I if and poly if it is integrable on both  $I_i = [a,c]$  and  $I_i = [c,b]$ . Prove also  $I_i = [c,b]$ .

If [I] is a sequence of continuous functions on a set  $A \subset B$  converging uniformly on A to a function  $f : A \to B$ , then f is continuous. Is the statement true if we replace uniform convergence by pointwise convergence Y

32. Prove that every non-empty open set on the vest line is the union of a opentable disport class of open intervals.

35. State and prove Baire's theorem.

Let 1° X -- Y, be a mapping of one topological space into another. Show that the following are equivalent.

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Reg. No. :	10. State the Cauchy Criterion for Uniform Com-
Name :	

Sixth Semester B.Sc. Degree (CBCSS-Reg./Supple./Improve.)

Examination, April 2021

(2014 – 2018 Admissions)

CORE COURSE IN MATHEMATICS

6B 13 MAT – Mathematical Analysis and Topology

Time: 3 Hours Max. Marks: 48

## SECTION - A

Answer all the questions. Each question carries 1 mark.

1. Let F, G be differentiable on [a,b] and let f = F' and g = G' belongs to R[a,b]. Then  $\int_a^b (fG + Fg) =$ 

- 2. Determine the radius of convergence of the power series  $\sum \frac{n^n}{n!} x^n$ .
- 3. Define topological space.
- 4. Fill in the blanks : The closure of the Cantor set is \_\_\_\_\_

## SECTION - E

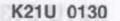
Answer any eight questions. Each question carries 2 marks.

- 5. Show that a constant function is Riemann integrable.
- 6. Using the Riemann Criterion for integrability, evaluate \int\_0^1 xdx.
- If f is continuous on [a,b] and let p be integrable on [a,b] and such that p(x) ≥ 0 for all x∈[a,b], show that there exist c ∈ [a,b] such that

$$\int_a^b f(x)p(x)dx = f(c)\int_a^b p(x)dx .$$

- 8. State Darboux's Theorem.
- Define pointwise convergence and uniform convergence of a sequence of functions.

P.T.O.



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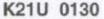
- 10. State the Cauchy Criterion for Uniform Convergence.
- 11. Prove that if f and  $g \in R[a,b]$ , then the product  $fg \in R[a,b]$ .
- 12. Let  $f: [a,b] \to R$  be integrable on [a,b]. If  $f(x) \ge 0$  for all  $x \in [a,b]$ . Is it true that  $\int_a^b f \ge 0$ ? Justify.
- Discuss the convergence of sequence (x<sup>n</sup>) for x∈ R.
- 14. If T, and T₂ are two topologies on X then prove that T₁∩ T₂ is a topology on X.
- 15. Show that the series  $\cos x + \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} + \frac{\cos 4x}{4^2} + \dots$  converges uniformly.
- 16. Prove that d(x,y) = |x y| is a metric on R.
- 17. Define complete metric space. Prove that (0,1) is not complete with Euclidean metric.
- 18. If X is a topological space and  $A \subseteq X$ . Show that  $\overline{A} = A \cup D(A)$ , where D (A) is the set of all limit points of A.
- 19. State Kuratowski closure axioms on topological space.
- 20. When a subset A of X is said to be nowhere dense in X? Give an example.

## SECTION - C

Answer any four questions. Each question carries 4 marks.

- 21. Let  $f : [a,b] \to R$  be bounded and let k < 0. Prove that L(kf) = kU(f) and U(kf) = kL(f).
- 22. Let  $f: [a,b] \to R$  be integrable on [a,b]. Prove that  $\left| \int_a^b f \right| \le k(b-a)$ , where  $|f(x)| \le k$ .
- 23. Show that if  $f: [a,b] \to R$  is continuous on [a,b], then f is integrable on [a,b].
- 24. Prove that a sequence  $(f_n)$  of bounded functions on  $A \subseteq R$  converges uniformly on A to f if and only if  $\|f_n f\|_A \to 0$ .
- 25. If R is the radius of convergence of the power series  $\sum (a_n x^n)$ , then prove that the series is absolutely convergent if |x| < R and divergent if |x| > R.





- Prove that a subspace Y of a complete metric space X is complete if and only if it is closed.
- 27. Let X and Y be metric spaces and  $f: X \to Y$ . If f is continuous at  $x_0$  then prove that  $x_n \to x_0 \Rightarrow f(x_n) \to f(x_0)$ .
- 28. Let X be a topological space and A is an arbitrary subset of X. Then prove that  $\bar{A} = \{x : \text{each neighborhood of x intersets A}\}.$

## SECTION - D

Answer any two questions. Each question carries 6 marks.

- 29. State and prove the Fundamental Theorem of Calculus (First Form).
- 30. Let I = [a,b] and let  $c \in (a,b)$ . Let  $f : I \to R$  be a bounded function. Then prove that f is integrable on I if and only if it is integrable on both  $I_1 = [a,c]$  and  $I_2 = [c,b]$ . Prove also  $\int_a^b f = \int_a^c f + \int_c^b f$ .
- 31. Prove the following:
  - If  $\{f_n\}$  is a sequence of continuous functions on a set  $A \subseteq R$  converging uniformly on A to a function  $f: A \to R$ , then f is continuous. Is the statement true if we replace uniform convergence by pointwise convergence?
- Prove that every non-empty open set on the real line is the union of a countable disjoint class of open intervals.
- 33. State and prove Baire's theorem.
- 34. Let f: X → Y be a mapping of one topological space into another. Show that the following are equivalent.
  - i) f is continuous
  - ii) f-1 (F) is closed in X.
  - iii)  $f(A) \subseteq \overline{f(A)}$