K21U 0130

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (CBCSS-Reg./Supple./Improve.)

Examination, April 2021

(2014 – 2018 Admissions)

CORE COURSE IN MATHEMATICS

6B 13 MAT – Mathematical Analysis and Topology

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **1** mark.

1. Let F, G be differentiable on [a,b] and let $f = F'$ and $g = G'$ belongs to R[a,b]. Then $\int_a^b (fG + Fg) =$
2. Determine the radius of convergence of the power series $\sum \frac{n^n}{n!} x^n$.
3. Define topological space.
4. Fill in the blanks : The closure of the Cantor set is __________

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

5. Show that a constant function is Riemann integrable.
6. Using the Riemann Criterion for integrability, evaluate $\int_0^1 x\,dx$.
7. If f is continuous on [a,b] and let p be integrable on [a,b] and such that $p(x) \geq 0$ for all $x \in [a,b]$, show that there exist $c \in [a,b]$ such that $\int_a^b f(x)p(x)dx = f(c)\int_a^b p(x)dx$.
8. State Darboux's Theorem.
9. Define pointwise convergence and uniform convergence of a sequence of functions.

P.T.O.

10. State the Cauchy Criterion for Uniform Convergence.

11. Prove that if f and g $\in$ R[a,b], then the product fg $\in$ R[a,b].

12. Let $f : [a,b] \to R$ be integrable on [a,b]. If $f(x) \geq 0$ for all $x \in [a,b]$. Is it true that $\int_a^b f \geq 0$? Justify.

13. Discuss the convergence of sequence (x^n) for $x \in R$.

14. If T_1 and T_2 are two topologies on X then prove that $T_1 \cap T_2$ is a topology on X.

15. Show that the series $\cos x + \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} + \frac{\cos 4x}{4^2} + \ldots$ converges uniformly.

16. Prove that $d(x,y) = |x - y|$ is a metric on R.

17. Define complete metric space. Prove that (0,1) is not complete with Euclidean metric.

18. If X is a topological space and $A \subseteq X$. Show that $\bar{A} = A \cup D(A)$, where D (A) is the set of all limit points of A.

19. State Kuratowski closure axioms on topological space.

20. When a subset A of X is said to be nowhere dense in X ? Give an example.

SECTION – C

Answer **any four** questions. **Each** question carries **4** marks.

21. Let $f : [a,b] \to R$ be bounded and let $k < 0$. Prove that $L(kf) = kU(f)$ and $U(kf) = kL(f)$.

22. Let $f : [a,b] \to R$ be integrable on [a,b]. Prove that $\left|\int_a^b f\right| \leq k(b-a)$, where $|f(x)| \leq k$.

23. Show that if $f : [a,b] \to R$ is continuous on [a,b], then f is integrable on [a,b].

24. Prove that a sequence (f_n) of bounded functions on $A \subseteq R$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$.

25. If R is the radius of convergence of the power series $\sum(a_n x^n)$, then prove that the series is absolutely convergent if $|x| < R$ and divergent if $|x| > R$.

26. Prove that a subspace Y of a complete metric space X is complete if and only if it is closed.

27. Let X and Y be metric spaces and $f : X \to Y$. If f is continuous at x_0 then prove that $x_n \to x_0 \Rightarrow f(x_n) \to f(x_0)$.

28. Let X be a topological space and A is an arbitrary subset of X. Then prove that $\bar{A}$ = {x : each neighborhood of x intersects A}.

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks.

29. State and prove the Fundamental Theorem of Calculus (First Form).

30. Let $I = [a,b]$ and let $c \in (a,b)$. Let $f : I \to R$ be a bounded function. Then prove that f is integrable on I if and only if it is integrable on both $I_1 = [a,c]$ and $I_2 = [c,b]$. Prove also $\int_a^b f = \int_a^c f + \int_c^b f$.

31. Prove the following :

    If $\{f_n\}$ is a sequence of continuous functions on a set $A \subseteq R$ converging uniformly on A to a function $f : A \to R$, then f is continuous. Is the statement true if we replace uniform convergence by pointwise convergence ?

32. Prove that every non-empty open set on the real line is the union of a countable disjoint class of open intervals.

33. State and prove Baire's theorem.

34. Let $f : X \to Y$ be a mapping of one topological space into another. Show that the following are equivalent.

    i) f is continuous

    ii) $f^{-1}(F)$ is closed in X.

    iii) $f(\bar{A}) \subseteq \overline{f(A)}$