K19U 0431

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Supplementary)
Examination, April 2019
(2013 and Earlier Admission)
CORE COURSE IN MATHEMATICS
6B10 MAT : Analysis and Topology

Time : 3 Hours   Max. Weightage: 30

1. Fill in the blanks.

   a) If $\mathcal{P} = (0, 0.5, 2.5, 3.5, 4)$ is a partition of $[0, 4]$, then $||\mathcal{P}|| =$ __________

   b) Let $A = [0, 1]$ and $f(x) = \frac{x}{5}$ for all $x \in A$. Then $|| f ||_A =$ __________

   c) Suppose that A is an open subset of a metric space X. Then Int (A) = __________

   d) If F denotes the Cantor set, then its closure $\bar{F} =$ __________

(Weightage 1)

Answer **any six** from the following. **Each** carries a Weightage 1.

2. Define tagged partition of a closed bounded interval in $\mathbb{R}$.

3. Prove that every constant function on [a, b] is Riemann integrable on [a, b].

4. Find the limit of the sequence of functions (f_n) where $f_n(x) = \frac{x^2 + nx}{n}$, $x \in \mathbb{R}$.

5. Let (f_n), (g_n) be sequences of bounded functions on A that converge uniformly on A to f, g respectively. Show that $(f_n g_n)$ converges uniformly on A to fg.

6. Prove that in every metric space X, the empty set ϕ and the full space X are open.

7. Define closed sphere in a metric space X. Give an example.

8. Show that in a metric space every convergent sequence is a Cauchy sequence.

9. Write two topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ on X = {a, b, c} so that $\mathcal{T}_1 \cup \mathcal{T}_2$ is not a topology.

10. Define a separable topological space and give an example.

(Weightage 6×1=6)

Answer **any seven** from the following. **Each** carries a Weightage **2**.

11. Show that the function $f(x) = \begin{cases} x+1, & \text{when } x \in [0,1] \text{ is rational} \\ 0, & \text{when } x \in [0,1] \text{ is irrational} \end{cases}$ is not Riemann integrable on [0, 1].

12. Prove that if $f : [a, b] \to \mathbb{R}$ is continuous on [a, b], then $f \in \mathcal{R}[a,b]$.

13. Show that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$.

14. Show that the sequence $\left(\frac{x^n}{1+x^n}\right)$ does not converge uniformly on [0, 2].

15. If $0 < R < \infty$ is the radius of convergence of a power series $\sum a_n x^n$, then prove that the series is absolutely convergent when $|x| < R$ and is divergent when $|x| > R$.

16. Let (X, d) be a metric space and $d_1(x, y) = \frac{d(x,y)}{1+d(x,y)}$, $\forall x, y \in X$. Then show that d_1 is also a metric on X.

17. Prove that a subset F of a metric space X is closed if and only if its complement F' is open.

18. If a convergent sequence in a metric space has infinitely many distinct points, then prove that its limit is a limit point of the set of points of the sequence.

19. Define subspace of a topological space and show that the subspace of a topological space is also a topological space.

20. Show that a subset of a topological space is closed if and only if it contains its boundary.

(Weightage 7×2=14)

Answer **any three** from the following. **Each** carries a Weightage **3**.

21. Prove that if $f, g \in \mathcal{R}[a,b]$ and $k \in \mathbb{R}$, then kf, $f+g \in \mathcal{R}[a, b]$. Also prove that in this case $\int_a^b kf = k\int_a^b f$ and $\int_a^b (f+g) = \int_a^b f + \int_a^b g$.

22. State and prove the Fundamental Theorem of Calculus (First form). Using the theorem evaluate $\int_a^b \frac{1}{x^2+1}\,dx$.

23. Let (f_n) be a sequence of functions in $\mathcal{R}[a, b]$ and suppose that (f_n) converges uniformly on [a, b] to f. Then prove that $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \lim_{n\to\infty} \int_a^b f_n$.

24. Define nowhere dense subset of a metric space and give an example. Prove that if a complete metric space is the union of a sequence of its subsets, then the closure of atleast one set in the sequence must have non-empty interior.

25. If A and B are subsets of a topological space X, then prove that :

i) $\bar{A}$ is the smallest closed set containing A,

ii) $\bar{\phi} = \phi$

iii) $\bar{\bar{A}} = \bar{A}$ and

iv) $\overline{A \cup B} = \bar{A} \cup \bar{B}$.

(Weightage 3×3=9)