K19U 0125

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.) Examination, April 2019
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B13MAT : Mathematical Analysis and Topology

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first 4 questions are **compulsory**. They carry **1** mark **each**.

1. Define the Riemann sum of a function $f : [a, b] \to \mathbb{R}$ corresponding to a tagged partition $\dot{P} = \{([x_i - 1, x_i], t_i)\}_{i=1}^{n}$.

2. Find the radius of convergence of $\sum \frac{x^n}{n}$.

3. State True or False: The subspace (0, 1] of $\mathbb{R}$ with usual metric is a complete metric space.

4. Suppose that T is the discrete topology on X = {a, b, c, d} and A = {b, c}. Then find Int(A).

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. If $f \in R[a, b]$ and $|f(x)| \le M$ for all $x \in [a, b]$, then show that $\left|\int_a^b f\right| \le M(b - a)$.

6. Show that Thomae's function, $f : [0, 1] \to \mathbb{R}$ given below is Riemann integrable over [0, 1].

$$f(x) = \begin{cases} 0, \text{ when } x \text{ is irrational} \\ 1, \text{ when } x = 0 \\ \frac{1}{n}, \text{ when } x = \frac{m}{n} \text{ is rational and is in the lowest form} \end{cases}$$

P.T.O.

7. Prove that if f and g belong to R[a, b], then the product fg belongs to R[a, b].

8. Test the uniform convergence of the sequence of functions, $f_n(x) = \frac{x}{n}$, $n \in \mathbb{N}$ on [0, 1].

9. Prove that if a sequence of continuous functions (f_n) defined on $A \subseteq \mathbb{R}$ converges uniformly on A to a function f, then f is continuous on A.

10. Show that in a metric space each open sphere is an open set.

11. Describe the Cantor set and show that it is closed in $\mathbb{R}$.

12. Prove that if a convergent sequence in a metric space has infinitely many distinct points, then its limit is a limit point of the set of terms of the sequence.

13. Prove that in the class of all topological spaces the relation, ~ defined by X ~ Y iff X and Y are homeomorphic is an equivalence relation.

14. Is the union of two topologies on a set a topology ? Justify.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry 4 marks **each**.

15. Show that if $f : [a, b] \to \mathbb{R}$ is monotone on [a, b], then $f \in R[a, b]$.

16. Using the substitution theorem evaluate $\int_1^4 \frac{\sin\sqrt{t}}{\sqrt{t}}\, dt$.

17. State and prove Cauchy criterion for uniform convergence.

18. Show that in a metric space X any finite intersection of open subsets of X is open in X. Give an example to show that in a metric space, a countable intersection of open sets need not be open.

19. Define the closure of a set in a metric space, give an example and show that closure of a set A is the smallest closed set containing A.

20. Let $f : X \to Y$ be a mapping of one topological space into another. Show that f is continuous if and only if $f^{-1}(F)$ is closed in X whenever F is closed in Y.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Prove that if $f, g : [a, b] \to \mathbb{R}$ are Riemann integrable on [a, b], then f + g is also integrable on [a, b].

22. If $f_n : [a, b] \to \mathbb{R}$ are Riemann integrable over [a, b] for every $n \in \mathbb{N}$ and $\sum f_n$ converges to f uniformly on [a, b], then show that f is Riemann integrable and $\int_a^b f = \sum_{n=1}^{\infty} \int_a^b f_n$.

23. If $\{A_n\}$ is a sequence of nowhere dense subsets in a complete metric space X, then prove that there exists a point in X which is not in any of the A_n's.

24. Let X be a non-empty set and C be a class of subsets of X which is closed under the formation of arbitrary intersections and finite unions. Prove that there exists a topology on X such that the class of all closed subsets of the space X coincides with C.

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