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THALASSE

K19U 0125

Reg. No. :

Name :

VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.)

Examination, April 2019

(2014 Admission Onwards)

CORE COURSE IN MATHEMATICS

6B13MAT : Mathematical Analysis and Topology

Time : 3 Hours

Max. Marks: 48

SECTION - A

All the first 4 questions are compulsory. They carry 1 mark each.

- 1. Define the Riemann sum of a function $f:[a,b]\to\mathbb{R}$ corresponding to a tagged partition $\dot{P}=\left\{\left(\left[x_i-1,x_i\right],t_i\right)\right\}_{i=1}^n$.
- 2. Find the radius of convergence of $\sum \frac{x^n}{n}$.
- State True or False: The subspace (0, 1] of ℝ with usual metric is a complete metric space.
- Suppose that T is the discrete topology on X = {a, b, c, d} and A = {b, c}. Then find Int(A).

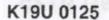
SECTION - B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

- $5. \ \ \text{If } f \in \ \text{R[a, b] and } |f(x)| \leq M \text{ for all } x \in [a, b], \text{ then show that } \left| \int_a^b f \right| \leq M(b-a).$
- 6. Show that Thomae's function, $f:[0,1]\to\mathbb{R}$ given below is Riemann integrable over [0,1].

$$f(x) = \begin{cases} 0, & \text{when } x \text{ is irrational} \\ 1, & \text{when } x = 0 \\ \frac{1}{n}, & \text{when } x = \frac{m}{n} \text{ is rational and is in the lowest form} \end{cases}$$

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- 7. Prove that if f and g belong to R[a, b], then the product fg belongs to R[a, b].
- 8. Test the uniform convergence of the sequence of functions, $f_n(x) = \frac{x}{n}$, $n \in \mathbb{N}$ on [0, 1].
- 9. Prove that if a sequence of continuous functions (f_n) defined on $A \subseteq \mathbb{R}$ converges uniformly on A to a function f, then f is continuous on A.
- 10. Show that in a metric space each open sphere is an open set.
- 11. Describe the Cantor set and show that it is closed in R.
- Prove that if a convergent sequence in a metric space has infinitely many distinct points, then its limit is a limit point of the set of terms of the sequence.
- Prove that in the class of all topological spaces the relation, ~ defined by X ~ Y
 iff X and Y are homeomorphic is an equivalence relation.
- 14. Is the union of two topologies on a set a topology? Justify.

SECTION - C

Answer any 4 questions from among the questions 15 to 20. These questions carry 4 marks each.

- 15. Show that if $f : [a, b] \to \mathbb{R}$ is monotone on [a, b], then $f \in R [a, b]$.
- 16. Using the substitution theorem evaluate $\int_1^4 \frac{\sin \sqrt{t}}{\sqrt{t}} dt$.
- 17. State and prove Cauchy criterion for uniform convergence.
- 18. Show that in a metric space X any finite intersection of open subsets of X is open in X. Give an example to show that in a metric space, a countable intersection of open sets need not be open.
- Define the closure of a set in a metric space, give an example and show that closure of a set A is the smallest closed set containing A.
- Let f: X → Y be a mapping of one topological space into another. Show that f is continuous if and only if f⁻¹ (F) is closed in X whenever F is closed in Y.



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SECTION - D

Answer any 2 questions from among the questions 21 to 24. These questions carry 6 marks each.

- 21. Prove that if f, g: [a, b] $\rightarrow \mathbb{R}$ are Riemann integrable on [a, b], then f + g is also integrable on [a, b].
- 22. If $f_n:[a,b]\to\mathbb{R}$ are Riemann integrable over [a,b] for every $n\in\mathbb{N}$ and $\sum f_n$ converges to f uniformly on [a,b], then show that f is Riemann integrable and $\int_a^b f = \sum_{n=1}^\infty \int_a^b f_n \ .$
- If {A_n} is a sequence of nowhere dense subsets in a complete metric space X, then prove that there exists a point in X which is not in any of the A'_ns.
- 24. Let X be a non-empty set and C be a class of subsets of X which is closed under the formation of arbitrary intersections and finite unions. Prove that there exists a topology on X such that the class of all closed subsets of the space X coincides with C.