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Name :

VI Semester B.Sc. Degree (CBCSS - Regular) Examination, May 2017 CORE COURSE IN MATHEMATICS (2014 Admn.)

6B13 MAT: Mathematical Analysis and Topology

Time: 3 Hours

Max. Marks: 48

SECTION - A

All the first 4 questions are compulsory. They carry 1 mark each.

- 1. Give an example of a function $f:[0,1] \to \mathbb{R}$ that is in R [c, 1] for every $c \in (0,1)$ but which is not in R [0, 1].
- 2. Find $\lim_{n\to\infty} \frac{x^n}{1+x^n}$ for $x \in \mathbb{R}$, $x \ge 0$.
- 3. Let d be the discrete metric on a set X which contains at least two points. Then for $x \in X$, what is the diameter of the open sphere $S_{1/2}(x)$?
- Give an example of a Cauchy sequence in a metric space X that does not converge in X.

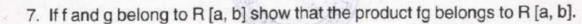
SECTION-B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

- $5. \text{ If } f \in R \text{ [a, b] and } \left|f(x)\right| \leq M \text{ for all } x \in \left[a, b\right] \text{ show that } \left|\int_a^b f\right| \leq M \big(b-a\big).$
- 6. If $f \in R[a, b]$ show that F defined by, $F(z) = \int_a^z f$ for $z \in [a, b]$, is continuous on [a, b].

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- 8. Show that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n f\|_{\Delta} \to 0$.
- 9. State and prove Weierstrass M-test for uniform convergence.
- Let X be a metric space. Show that every subset of X is open
 ⇔ each subset of X which consists of a single point is open.
- 11. Write a short note on the Cantor set.
- 12. Show that in any metric space, each closed sphere is a closed set.
- Show that the union of two topologies on a nonempty set X need not be a topology on X.
- 14. Prove or disprove : If A and B are subsets of a topological space X with $\overline{A} = \overline{B}$, then A = B. (2x8=16)

SECTION - C

Answer any 4 questions from among the questions 15 to 20. These questions carry 4 marks each.

- 15. If f ∈ R [a, b], show that f is bounded on [a, b].
- 16. Suppose that f is continuous on [a, b], that $f(x) \ge 0$ for all $x \in [a, b]$ and that $\int_a^b f = 0$. Prove that f(x) = 0 for all $x \in [a, b]$. Can the continuity hypothesis be dropped? Justify.
- 17. Let $f_n(x) = \frac{nx}{1 + nx}$ for $x \in [0, 1]$.
 - a) Evaluate $\lim_{n\to\infty} \int_0^1 f_n(x) dx$
- b) Find the pointwise limit function f
- c) Evaluate $\int_0^1 f(x) dx$
- d) Does (fn) converge uniformly to f?

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18. Let X be a metric space with metric d. Show that d_1 defined by $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)} \text{ is also a metric on X}.$

- 19. Let X be a topological space and A a subset of X. Show that
 - i) $\overline{A} = A \cup D(A)$ and
- ii) A is closed $\Leftrightarrow A \supseteq D(A)$
- 20. Let $X = \{1, 2, 3\}$ and with the topology $T = \{X, \emptyset, \{1\}, \{1, 2\}, \{1, 3\}\}$.
 - a) List all closed subsets of X

c) Find the closure of {2}

- b) Find the closure of {1}
- d) List all open subsets of X

(4×4=16)

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SECTION-D

Answer any 2 questions from among the questions 21 to 24. These questions carry 6 marks each.

- 21. State and prove the fundamental theorem of calculus (first form).
- 22. Show that the uniform convergence of the sequence of continuous functions is sufficient to guarantee the continuity of the limit function. Is it necessary? Justify.
- 23. State and prove Cantor's intersection theorem.
- State the Kuratowski closure axioms on a non-empty set X and show that it defines a topology on X. (6x2=12)