Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (CBCSS – Reg./Supple./Improve.) Examination, April 2021
(2014 – 2018 Admissions)
CORE COURSE IN MATHEMATICS
6B10 MAT : Linear Algebra

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **1** mark.

1. Define a vector space V over a field F.
2. When we say V is the direct sum of the subspaces W_1 and W_2 ?
3. Find the characteristic roots of $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 3 \\ 0 & 0 & 4 \end{pmatrix}$.
4. Show that the vectors starting from the origin and terminating at (–3, 1, 7) and (9, –3, –21) are parallel.

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

5. Let W_1, W_2 are two subspaces of a vector space V then prove that $W_1 \cap W_2$ is a subspace.
6. Show that $2x^3 - 2x^2 + 12x - 6$ is a linear combination of $x^3 - 2x^2 - 5x - 3$ and $3x^3 - 5x^2 - 4x - 9$.
7. Prove Cancellation law of vector addition.
8. Let V and W be vector spaces and $T : V \to W$ be linear then show that N(T) is a subspace of V.

9. Let V and W be vector spaces and T from V to W be linear then prove that T is one to one if and only if $N(T) = \{0\}$.

10. Show that $T : R^2 \to R^2$ by $T(a_1, a_2) = (2a_1 + a_2, a_1)$ is linear.

11. Let A be an $m \times n$ matrix and let B and C be $n \times p$ matrices then prove that $A(B + C) = AB + AC$.

12. Find the product of characteristic roots of $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$.

13. Find the characteristic equation of $A = \begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}$.

14. Define eigenvalue and eigenvector of a Matrix.

15. Show that A and A′ have the same eigenvalues.

16. Show that every singular matrix is a right as well as left zero divisor.

17. If X_1, X_2 are solutions of $AX = 0$, then show that $k_1X_1 + k_2X_2$ is also a solution. Where k_1, k_2 are scalars.

18. Find the equation of line through (–2, –1, 5) and (9, –3, –21).

19. Use Gaussian elimination method, solve $2x + y + z = 10$; $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$.

20. Show that $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ is diagonalizable.

SECTION – C

Answer **any four** questions. **Each** question carries **4** marks.

21. Let S be a linearly independent subset of a vector space V, and let x be an element of V that is not in S. Then prove that $S \cup \{x\}$ is linearly dependent if and only if $x \in \text{Span}(S)$.

22. Let V be a vector space and S a subset that generates V. If B is a maximal linearly independent subset of S, then show that B is a basis for V.

23. Define $U : R^2 \to R^3$ by $U(a_1, a_2) = (a_1 - a_2, 2a_1, 3a_1 + 2a_2)$. Find the matrix of U with respect to standard ordered basis.

24. Prove that every square matrix satisfies its characteristic equations.

25. If A is non singular, prove that the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A.

26. Check the consistency and solve $x + y + z = 6$; $x - y + z = 2$; $2x + y - z = 1$.

27. Use Gauss Jordan method, solve $5x - 2y + z = 4$; $7x + y - 5z = 8$; $3x + 7y + 4z = 10$.

28. Find the characteristic roots and the corresponding characteristic vectors of the matrix $\begin{Bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{Bmatrix}$.

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks.

29. If S is a non empty subset of a vector space V, then show that the set W consist of all linear combinations of elements of S is a subspace of V. Moreover W is the smallest subspace of V containing S.

30. Find a bases for the subspaces $W_1 = \{(a_1, a_2, a_3, a_4, a_5) \in F^5 / a_1 - a_3 - a_4 = 0\}$ and $W_2 = \{(a_1, a_2, a_3, a_4, a_5) \in F^5 / a_2 = a_3 = a_4 = 0, a_1 + a_5 = 0\}$ of F^5. What are the dimension of W_1 and W_2 ?

31. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$. Show that for every integer $n \geq 4$, $A^n = A^{n-2} + A^3 - A$. Hence evaluate A^{20}.

32. Find all latent vectors of the matrix $\begin{pmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$.

33. Use Gauss method compute the inverse of $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$.

34. Investigate for what value of λ, μ, the system of equation $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda z = \mu$ has (i) No solution, (ii) a unique solution, (iii) a infinite number of solutions.
