



22. State and prove dimension theorem. Deduce that a linear transformation  $T: V \rightarrow V$  is one to one if and only if  $T$  is onto.

23. Show that the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

satisfies Cayley Hamilton theorem and hence obtain  $A^{-1}$ .

24. Prove that

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

is diagonalizable and find the diagonal form.



Reg. No. : .....

Name : .....

VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.)

Examination, April 2020

(2014 Admission Onwards)

CORE COURSE IN MATHEMATICS

6B10MAT : Linear Algebra

Time : 3 Hours

Max. Marks : 48

SECTION - A

All the first 4 questions are compulsory. They carry 1 mark each.

1. Give an example to show that if  $f$  and  $g$  are two quadratic polynomials then the polynomial  $f + g$  need not be quadratic.
2. Obtain a basis for  $M_{2 \times 2}(\mathbb{R})$ .
3. Let  $V = P_2(\mathbb{R})$  and let  $\beta = \{1, x, x^2\}$  be the standard ordered basis for  $V$ . If  $f(x) = 3x^2 + 2x + 1$  then  $[f]_\beta$  is
4. Give the nature of characteristic roots of
  - i) a Hermitian matrix and
  - ii) a Unitary matrix.

SECTION - B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Find the equation of the line through the points  $P(2, 0, 1)$  and  $Q(4, 5, 3)$ .
6. What is the possible difference between a generating set and a basis?
7. Is the union of two subspaces  $W_1$  and  $W_2$  of a vector space  $V$  again a subspace of  $V$ ? Justify with an example.



8. Let  $V$  be a vectorspace and  $\beta = \{x_1, x_2, \dots, x_n\}$  be a subset of  $V$ . Show that  $\beta$  is basis if each vector  $y$  in  $V$  can be uniquely expressed as a linear combination of vectors in  $\beta$ .
9. Show that  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined by  $T(a_1, a_2) = (2a_1 + a_2, a_1)$  is a linear transformation.
10. Let  $T : V \rightarrow W$  be a linear transformation. Prove that  $N(T)$ , the nullspace of  $T$ , is a subspace of  $V$ .

11. Find a basis of the row space of the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix}$$

12. Find the characteristic values of the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$$

13. Use Gauss elimination to solve the system of equations :

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

$$x + y + 3z = 5.$$

14. Use Gauss, Jordan elimination to solve the system of equations :

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

$$x + y + 3z = 5.$$



## SECTION – C

Answer **any 4** questions from among the questions **15 to 20**. These questions carry **4 marks each**.

15. In every vectorspace  $V$  over a field  $F$  prove that

- i)  $a0 = 0 \forall a \in F$ , where  $0$  is the zero vector and  
 ii)  $(-a)x = -(ax) \forall a \in F$  and  $\forall x \in V$ .

16. Define linear dependence and linear independence of vectors with examples.

17. Define a linear transformation from a vectorspace  $V$  into  $W$ . Verify that  $T : M_{m \times n} \rightarrow M_{n \times m}$  by  $T(A) = A^t$  where  $A^t$  is the transpose of  $A$ , is linear.

18. Show that the row nullity and column nullity of a square matrix are equal.

19. Find the characteristic values and the corresponding characteristic vectors of the matrix.

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

20. Use the Gaussian elimination method to find the inverse of the matrix.

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$$

## SECTION – D

Answer **any 2** questions from among the questions **21 to 24**. These questions carry **6 marks each**.

21. If a vectorspace  $V$  is generated by a finite set  $S_0$ , then show that a subset of  $S_0$  is a basis for  $V$  and  $V$  has a finite basis.