



22. State and prove dimension theorem. Deduce that a linear transformation $T : V \rightarrow V$ is one to one if and only if T is onto.

23. Show that the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

satisfies Cayley Hamilton theorem and hence obtain A^{-1} .

24. Prove that

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

is diagonalizable and find the diagonal form.



Reg. No. :

Name :

VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.)

Examination, April 2020

(2014 Admission Onwards)

CORE COURSE IN MATHEMATICS

6B10MAT : Linear Algebra

Time : 3 Hours

Max. Marks : 48

SECTION - A

All the first 4 questions are compulsory. They carry 1 mark each.

1. Give an example to show that if f and g are two quadratic polynomials then the polynomial $f + g$ need not be quadratic.
2. Obtain a basis for $M_{2 \times 2}(R)$.
3. Let $V = P_2(R)$ and let $\beta = \{1, x, x^2\}$ be the standard ordered basis for V . If $f(x) = 3x^2 + 2x + 1$ then $[f]_\beta$ is
4. Give the nature of characteristic roots of
 - i) a Hermitian matrix and
 - ii) a Unitary matrix.

SECTION - B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Find the equation of the line through the points $P(2, 0, 1)$ and $Q(4, 5, 3)$.
6. What is the possible difference between a generating set and a basis?
7. Is the union of two subspaces W_1 and W_2 of a vector space V again a subspace of V ? Justify with an example.



8. Let V be a vectorspace and $\beta = \{x_1, x_2, \dots, x_n\}$ be a subset of V . Show that β is basis if each vector y in V can be uniquely expressed as a linear combination of vectors in β .
9. Show that $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(a_1, a_2) = (2a_1 + a_2, a_1)$ is a linear transformation.
10. Let $T : V \rightarrow W$ be a linear transformation. Prove that $N(T)$, the nullspace of T , is a subspace of V .

11. Find a basis of the row space of the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix}$$

12. Find the characteristic values of the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$$

13. Use Gauss elimination to solve the system of equations :

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

$$x + y + 3z = 5.$$

14. Use Gauss, Jordan elimination to solve the system of equations :

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

$$x + y + 3z = 5.$$



SECTION – C

Answer **any 4** questions from among the questions 15 to 20. These questions carry **4** marks each.

15. In every vectorspace V over a field F prove that

i) $a0 = 0 \forall a \in F$, where 0 is the zero vector and

ii) $(-a)x = -(ax) \forall a \in F$ and $\forall x \in V$.

16. Define linear dependence and linear independence of vectors with examples.

17. Define a linear transformation from a vectorspace V into W . Verify that $T : M_{m \times n} \rightarrow M_{n \times m}$ by $T(A) = A^t$ where A^t is the transpose of A , is linear.

18. Show that the row nullity and column nullity of a square matrix are equal.

19. Find the characteristic values and the corresponding characteristic vectors of the matrix.

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

20. Use the Gaussian elimination method to find the inverse of the matrix.

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$$

SECTION – D

Answer **any 2** questions from among the questions 21 to 24. These questions carry **6** marks each.

21. If a vectorspace V is generated by a finite set S_0 , then show that a subset of S_0 is a basis for V and V has a finite basis.