



K19U 0122

Reg. No. :

Name :

VI Semester B.Sc. Degree (CBCSS-Reg/Supple/Improv.) Examination, April 2019
 (2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B10 MAT : Linear Algebra

Time : 3 Hours

Max. Marks : 48

SECTION - A

All the first 4 questions are compulsory. They carry 1 mark each.

1. Give an example of a proper non trivial subspace of $P(R)$, the vectorspace of all polynomials with real coefficients.
2. A subset of a linearly dependent set can possibly be linearly independent. Justify by giving an example.
3. The null space of the operator $T : R^2 \mapsto R^2$ given by $T(a_1, a_2) = (a_1, 0)$ is
4. The number of linearly independent solutions of the equation $x + y + z = 0$ is

SECTION - B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. In any vectorspace V show that $(a + b)(x + y) = ax + bx + ay + by$ for all scalars a and b and all vectors x and y .
6. Let $V = R^2 = R \times R$ where vector addition and scalar multiplication are defined by; $(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$ and $r(x_1, x_2) = (rx_1, x_2)$. Is V a vectorspace over R ? Justify.
7. Show that any intersection of subspaces of a vectorspace V is again a subspace of V .
8. Let $T : V \mapsto W$ be a linear transformation. Prove that $N(T)$, the nullspace of T , is a subspace of V .
9. Let $T : V \mapsto W$ be an invertible linear transformation. Use dimension theorem to observe that $\dim V = \dim W$.

P.T.O.



10. Let $V = C[a,b]$ be the vectorspace of all continuous real valued functions defined over the closed bounded interval $[a, b]$. Describe the fundamental theorem of calculus in terms of linear transformations on V .
11. Find the values of λ for which the following system of equations have non zero solutions.
 $\lambda x + 8y = 0$
 $2x + \lambda y = 0$
12. Verify that the set of all characteristic vectors of a square matrix associated with a fixed characteristic value λ is a subspace of the respective Euclidean space.
13. Use Gauss elimination to solve the system of equations :
 $2x + y + z = 10$
 $3x + 2y + 3z = 18$
 $x + 4y + 9z = 16$
14. Use Gauss Jordan elimination to solve the system of equations :
 $2x + y + z = 10$
 $3x + 2y + 3z = 18$
 $x + 4y + 9z = 16$

SECTION - C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Define vectorspace and show that in every vector space $(-1)x$ is the additive inverse of x .
16. Define a basis of a vectorspace. Give an example of a basis of $M_{2 \times 2}(\mathbb{R})$.
17. Let V and W be vectorspaces and let $T : V \rightarrow W$ be linear. Then prove that T is one to one if and only if $N(T) = \{0\}$.
18. Suppose that $AX = B$ has a solution. Show that this solution is unique if and only if $AX = 0$ has only the trivial solution.



19. Test the following system of equations for consistency and solve it if it is consistent.
 $x + 2y + 3z = 14$
 $3x + y + 2z = 11$
 $2x + 3y + z = 11$
20. Find the largest characteristic value and a corresponding characteristic vector of the matrix.

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

SECTION - D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. If S is a nonempty subset of a vectorspace V , then show that $\text{span}(S)$ is a subspace of V and is the smallest subspace of V containing S . Under what further condition S can become a basis of V ?
22. Let V and W be vectorspaces over a common field F and suppose that V has a basis $\{x_1, x_2, \dots, x_n\}$. Prove that for any fixed vectors y_1, y_2, \dots, y_n in W there exists exactly one linear transformation $T : V \rightarrow W$ such that $T(x_i) = y_i$ for $i = 1 \dots n$.
23. Show that the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

satisfies Cayley Hamilton theorem and hence obtain A^{-1} .

24. Prove that

$$A = \begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

is diagonalizable and find the diagonal form.