K16U 0203

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2016
CORE COURSE IN MATHEMATICS
6B12 MAT : Linear Algebra

Time : 3 Hours | Max. Weightage : 30

1. Fill in the blanks :

   a) Dimension of trivial vector space {0} is ________

   b) The largest subspace of a vector space V is ________

   c) In a row reduced echelon matrix, if a column contains leading entry 1, then all other entries in that column are ________

   d) If T is a linear transformation, then the dimension of range space of T is known as ________ **(Weightage : 1)**

Answer **any six** from the following : **(Weightage : 1 each)**

2. What do you mean by span of a set ?

3. Give a basis for $\mathbb{C}$.

4. Prove that $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \,/\, x_1 + x_2 + x_3 = 0\}$ is a subspace of $V = \mathbb{R}^3$.

5. Using graphs, solve $2x + y = 3$; $4x + 2y = 6$.

6. What do you mean by row-rank of a matrix ?

7. If $\lambda (\neq 0)$ is an eigen value of a non-singular matrix A, prove that $\frac{1}{\lambda}$ is an eigen value of A^{-1}.

8. If $T : U \to V$ be a linear map, then prove that $T(0_U) = 0_V$.

P.T.O.

9. Define kernal of a linear transformation.

10. What do you mean by idempotent map ? Give an example. **(Weightage 6×1=6)**

Answer **any seven** from the following : **(Weightage 2 each)**

11. Let F be the set of all real valued functions from $\mathbb{R}$ into $\mathbb{R}$. Show that F is a vector space with respect to the operations.

$(f + g)(x) = f(x) + g(x)$; and $(\alpha f)(x) = \alpha f(x)\ \forall x \in \mathbb{R}$.

12. Determine whether or not the vectors (1, –1, 2), (2, 3, 1) and (4, 5, 6) in $\mathbb{R}^3$ are linearly dependent.

13. Show that the equations $2x - 3y + 4z = 23$, $3x + 4y - 8z = -19$, $4x - y - 2z = 11$, $x + 2y - 2z = -7$ are consistent and solve them.

14. Find the eigen values and eigen vector corresponding to the largest eigen value of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$.

15. Prove that eigen vectors corresponding to distinct eigen values of a square matrix are linearly independent.

16. Prove that constant term of the characteristic polynomial of a matrix A is $(-1)^n |A|$ where n is the order of A.

17. Check whether the function $T : P_2 \to \mathbb{R}^3$ defined by $T(a + bx + cx^2) = (c - a, a + b, b + c)$ is a linear transformation or not.

18. Find the null space, range space and their dimensions of the linear transformation $T : R^3 \to P_2$ defined by $T(a, b, c) = (a + c) + (b - a)x + (b + c)x^2$.

19. Let T be a linear operator defined on $\mathbb{R}^3$ such that $T(e_1) = e_1 + e_2$, $T(e_2) = e_2 + e_3$, $T(e_3) = e_1 + e_2 + e_3$ where $\{e_1, e_2, e_3\}$ is a standard basis for $\mathbb{R}^3$. Is T non-singular ? If so, find T^{-1}.

20. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}$. **(Weightage 7×2=14)**

Answer **any three** from the following : **(Weightage 3 each)**

21. Let $U = \{(x_1, x_2, x_3) \in \mathbb{R}^3 / x_1 + x_2 - 2x_3 = 0\}$ and $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 / x_1 - 3x_2 + 2x_3 = 0\}$ be subspaces of $\mathbb{R}^3$. Find a basis and dimension of U, W and $U \cap W$.

22. Using the row reduction method, check whether the given matrix $A = \begin{bmatrix} -1 & 1 & 1 \\ 3 & 1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$ is invertible or not. If it is invertible, find A^{-1}.

23. Using Cayley-Hamilton theorem, find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{bmatrix}$.

24. Diagonalise the matrix $A = \begin{bmatrix} -2 & 4 & -2 \\ 4 & 4 & -4 \\ -2 & -4 & 5 \end{bmatrix}$.

25. What do you mean by a matrix related to linear transformation ? Let $T : P_2 \to R^3$ be a linear transformation defined by $T(a + bx + cx^2) = (a, c - b, c - a)$. Find the matrix representation of T with respect to the ordered basis $B_1 = \{1, x, x^2\}$ of P_2 and $B_2 = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ of R^3. **(Weightage 3×3=9)**