Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Supple./Improv.) Examination, May 2017
CORE COURSE IN MATHEMATICS
6B13 MAT : Integral Transforms
(2009-2013 Admns.)

Time : 3 Hours

Max. Weightage : 30

1. Fill in the blanks :

   a) Laplace transform of t^2 is _____

   b) Fundamental period of cos2x is _____

   c) Product of an even and an odd function is _____

   d) Z (u(n)) = _____

   (Weightage : 1)

Answer **any six** from the following. (Weightage **1 each**) :

2. State the condition for the existence of Laplace transform.

3. Find $L((t+1)^2 e^t)$.

4. Find the inverse Laplace transform of $\dfrac{2}{s^2 + s + \dfrac{1}{2}}$.

5. Explain the Fourier sine series and cosine series expansion of functions.

6. State final value theorem for Z-transform.

7. Find Z-transform of $\cos \dfrac{n\pi}{2}$.

8. Find Z-transform of $(t + T)e^{-(t+T)}$.

9. Explain Fourier integral representation of functions.

10. Find Fourier cosine transform of $f(x) = e^{-x}$.

    (Weightage : 6×1=6)

Answer **any seven** from the following. (Weightage **2 each**) :

11. State and prove first shifting theorem for Laplace transform.

12. Find the inverse Laplace transform of $\log\left(1 + \dfrac{w^2}{s^2}\right)$.

P.T.O.

13. Find the Fourier series expansion of $f(x) = \pi x$ in the interval $0 < x < 2$.

14. Express $f(x) = t - t^2$ as a half range sine series in $0 < t < 1$.

15. Find the complex Fourier series of $f(x) = e^{-x}$, $-1 \leq x \leq 1$.

16. State and prove first shifting theorem for Z-transforms.

17. Find the Z-transform of $f * g$ where $f(n) = \cos\dfrac{n\pi}{2}$ and $g(n) = \sin\dfrac{n\pi}{2}$.

18. Using convolution method, find the inverse Z-transform of $\dfrac{z^2}{(z-2)(z-3)}$.

19. Find the Fourier integral of $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$.

20. With usual notation, prove that $F_s\{f''(x)\} = -w^2 F_s\{f(x)\} + \sqrt{\dfrac{2}{\pi}}\, wf(0)$.

(Weightage : 7×2=14)

Answer **any three** from the following. (Weightage **3 each**) :

21. Solve the integral equation $y(t) = t + \displaystyle\int_0^t y(\tau)\sin(t - \tau)d\tau$.

22. Using Laplace transform, solve the initial value problem :

$$\dfrac{dy}{dt} + 2x + y = 0;\ \dfrac{dx}{dt} + 5x - 2y = t,\ y(0) = x(0) = 0.$$

23. Obtain the Fourier series for the function $f(x) = x - x^2$ in the interval $(-\pi, \pi)$.

Deduce that $\dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \ldots = \dfrac{\pi^2}{12}$.

24. a) Find Z-transform of $\cos n\theta$.

b) Find the inverse Z-transform of $\dfrac{8z^2}{(2z-1)(4z-1)}$.

25. Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| < 1 \end{cases}$. Hence evaluate $\displaystyle\int_0^\infty \dfrac{\sin x}{x}\,dx$.

(Weightage : 3×3=9)