



Reg. No. : .....

Name : .....

**VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.)**

**Examination, May 2016**

**Core Course in Mathematics**

**6B13 MAT : INTEGRAL TRANSFORMS**

Time : 3 Hours

Max. Weightage : 30

1. Fill in the blanks :

- a) Laplace transform of  $t$  is \_\_\_\_\_
- b) A function without fundamental period is \_\_\_\_\_
- c) Example for an odd function is \_\_\_\_\_
- d)  $Z(k) =$  \_\_\_\_\_

**(Weightage 1)**

Answer **any six** from the following (Weightage **1 each**).

- 2. State linearity property of Laplace transform.
- 3. Find  $L(te^{-3t})$ .
- 4. Find the inverse Laplace transform of  $\frac{4}{s^2 - 2s - 3}$ .
- 5. Explain the convergence of Fourier series expansion of functions.
- 6. State second shifting theorem for Z-transform.
- 7. Find Z-transform of  $n(n-1)$ .
- 8. Find Z-transform of  $e^{2(t+T)}$ .
- 9. State Fourier integral theorem.
- 10. Find Fourier cosine transform of

$$f(x) = \begin{cases} x & 0 < x < a \\ 0 & x > a \end{cases}$$

**(Weightage 6x1=6)**



Answer **any seven** from the following (Weightage **2 each**).

11. Define Dirac's delta function. Also find its Laplace transform.
12. Find the inverse Laplace transform of  $\log \left( \frac{s+1}{s-1} \right)$ .
13. Find the Fourier series expansion of  $e^{-x}$  in the interval  $(-l, l)$ .
14. Express  $f(x) = x^2$  as a half range sine series in  $0 < x < \pi$ .
15. Obtain the complex form of the Fourier series formula.
16. State and prove convolution theorem for Z-transforms.
17. Find the Z-transform of  $f * g$  where  $f(n) = 3^n$  and  $g(n) = \cos n\theta$ .
18. Using power series method, find the inverse Z-transform of  $\log \left( \frac{z}{z+1} \right)$ .
19. Prove that  $\int_0^{\infty} \frac{w \sin xw}{k^2 + w^2} dw = \frac{\pi}{2} e^{-kx}$ ,  $x > 0, k > 0$ .
20. Let  $f(x)$  be continuous on the  $x$ -axis,  $f(x) \rightarrow 0$  as  $|x| \rightarrow \infty$  and  $f'(x)$  be absolutely integrable on the  $x$ -axis, then prove that  

$$F\{f''(x)\} = -w^2 F\{f(x)\}.$$
**(Weightage 7x2=14)**

Answer **any three** from the following (Weightage **3 each**).

21. If  $f(t)$  is a periodic function with period  $T$ , then prove that  

$$L(f(t)) = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt.$$
22. Using Laplace transform, solve the initial value problem:  
 $y''' - 3y'' + 3y' - y = t^2 e^t, y(0) = 1, y'(0) = 0, y''(0) = -2.$



23. Obtain the Fourier series for the function  $f(x) = \begin{cases} x, & 0 \leq x \leq \pi \\ 2\pi - x, & \pi \leq x \leq 2\pi \end{cases}$

$$\text{Deduce that } \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}.$$

24. a) Find Z-transform of  $r^n \sin n\theta$ .

- b) Find the inverse Z-transform of  $\frac{z^3 - 20z}{(z-2)^3(z-4)}$ .

25. Find the Fourier transform of  $f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$

$$\text{Hence evaluate } \int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right) dx.$$

**(Weightage 3x3=9)**