Reg. No. : ....

Name : ....

K17U 0369

VI Semester B.Sc. Degree (CBCSS – Regular) Examination, May 2017
CORE COURSE IN MATHEMATICS
(2014 Admn.)
6B12 MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Find the principal value of the argument of the complex number $-\pi - i\pi$.

2. Evaluate $\int_{-i}^{i} \frac{dz}{z}$.

3. Show by example that f being analytic is not necessary to hold $\oint_c f(z)\, dz = 0$.

4. When do you say that z_0 is an isolated singularity of f(z) ?

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Prove that :

   a) z is real if and only if $\overline{z} = z$.

   b) z is either real or pure imaginary if and only if $\overline{z}^2 = z^2$.

6. Show that an analytic function of constant absolute value is constant.

7. Find all values of $\sqrt[3]{216}$.

P.T.O.

8. Represent $12\left(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi\right)$ in the form x + iy and plot in the complex plane.

9. Determine whether the function f defined by $f(z) = \bar{z}$ is analytic.

10. Evaluate $\int_C \text{Re}\, z\, dz$, C the parabola $y = x^2$ from 0 to 1 + i.

11. Determine whether the series $\sum_{n=1}^{\infty} n^2 \left(\frac{i}{3}\right)^n$ is convergent or divergent.

12. Find the radius of curvature of the power series, $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2}(z-3i)^n$.

13. Find the Laurent series of $\frac{1}{z(z-1)}$ that converges for $0 < |z| < R$ and determine the precise region of convergence.

14. Show that the zeros of an analytic function f (z) ($\neq 0$) are isolated.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Find the principal value of $(1-i)^{1+i}$.

16. State and prove Cauchy's integral formula.

17. Integrate $g(z) = (z^2 - 1)^{-1} \tan z$ around the circle C : $|z| = 3/2$ (counter-clockwise).

18. Find the Maclaurin series of $f(z) = \tan^{-1} z$.

19. If a series $z_1 + z_2 + \ldots$ is such that $\lim_{n\to\infty} \sqrt[n]{|z_n|} = L$ then show that
    a) The series converges absolutely if $L < 1$.
    b) The series diverges if $L > 1$.

20. Determine the location and type of singularities of $\tan \frac{1}{2}\pi z$, including those at infinity. In the case of poles also state the order.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks each.

21. Show that
    a) $\cos z = \cos x \cosh y - i \sin x \sinh y$ and $\sin z = \sin x \cosh y + i \cos x \sinh y$
    b) $|\cos z|^2 = \cos^2 x + \sinh^2 y$ and $|\sin z|^2 = \sin^2 x + \sinh^2 y$.
    c) cos z and sin z are periodic with period 2π.

22. a) Integrate $\frac{\text{Ln}(z+3) + \cos z}{(z+1)^2}$ counter clockwise around the circle $|z| = 2$.
    b) State and prove Liouville's theorem.

23. Develop $\cos \pi z$ in a Taylor series with $\frac{1}{2}$ as center. Find the radius of convergence.

24. Evaluate the integral $\oint_C \left(\frac{ze^{\pi z}}{z^4 - 16} + ze^{\pi/z}\right) dz$ where C is the ellipse $9x^2 + y^2 = 9$, counter clockwise.