

K17U 0369

SECTION - A

Answer any 8 questions from among the questions 1 to 4. These questions carry 2 marks each.

1. Show that $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$ and $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$.
2. Prove that $\cos^2 x + \sin^2 x = 1$ and $\sin^2 x = 1 - \cos^2 x$.
3. Prove that $\cos^2 x - \sin^2 x = \cos 2x$ and $\sin^2 x - \cos^2 x = -\cos 2x$.
4. Prove that $\cos^2 x - \sin^2 x = \cos 2x$ and $\sin^2 x - \cos^2 x = -\cos 2x$.

5. Evaluate $\int \frac{1}{1+x^2} dx$.

6. Evaluate $\int \frac{1}{1-x^2} dx$.

7. Evaluate $\int \frac{1}{1+x^2} dx$.

8. Evaluate $\int \frac{1}{1-x^2} dx$.

9. Evaluate $\int \frac{1}{1+x^2} dx$.

10. Evaluate $\int \frac{1}{1-x^2} dx$.



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Reg. No. :

Name :

VI Semester B.Sc. Degree (CBCSS – Regular) Examination, May 2017
CORE COURSE IN MATHEMATICS
(2014 Admn.)
6B12 MAT : Complex Analysis

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the first 4 questions are **compulsory**. They carry 1 mark each.

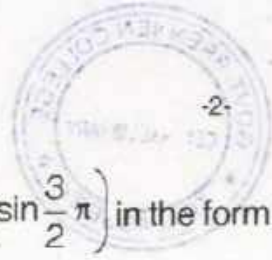
1. Find the principal value of the argument of the complex number $-\pi - i\pi$.
2. Evaluate $\int_{-1}^1 \frac{dz}{z}$.
3. Show by example that f being analytic is not necessary to hold $\oint_C f(z) dz = 0$.
4. When do you say that z_0 is an isolated singularity of $f(z)$?

SECTION – B

Answer **any 8** questions from among the questions 5 to 14. These questions carry **2 marks each**.

5. Prove that :
 - a) z is real if and only if $\bar{z} = z$.
 - b) z is either real or pure imaginary if and only if $\bar{z}^2 = z^2$.
6. Show that an analytic function of constant absolute value is constant.
7. Find all values of $\sqrt[3]{216}$.

P.T.O.



8. Represent $12\left(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi\right)$ in the form $x + iy$ and plot in the complex plane.

9. Determine whether the function f defined by $f(z) = \bar{z}$ is analytic.

10. Evaluate $\int_C \operatorname{Re} z \, dz$, C the parabola $y = x^2$ from 0 to $1 + i$.

11. Determine whether the series $\sum_{n=1}^{\infty} n^2 \left(\frac{i}{3}\right)^n$ is convergent or divergent.

12. Find the radius of curvature of the power series, $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} (z - 3i)^n$.

13. Find the Laurent series of $\frac{1}{z(z-1)}$ that converges for $0 < |z| < R$ and determine the precise region of convergence.

14. Show that the zeros of an analytic function $f(z) (\neq 0)$ are isolated.

SECTION - C

Answer any 4 questions from among the questions 15 to 20. These questions carry 4 marks each.

15. Find the principal value of $(1 - i)^{1+i}$.

16. State and prove Cauchy's integral formula.

17. Integrate $g(z) = (z^2 - 1)^{-1} \tan z$ around the circle $C : |z| = 3/2$ (counter-clockwise).

18. Find the Maclaurin series of $f(z) = \tan^{-1} z$.

19. If a series $z_1 + z_2 + \dots$ is such that $\lim_{n \rightarrow \infty} \sqrt[n]{|z_n|} = L$ then show that

- The series converges absolutely if $L < 1$.
- The series diverges if $L > 1$.

20. Determine the location and type of singularities of $\tan \frac{1}{2}\pi z$, including those at infinity. In the case of poles also state the order.



SECTION - D

Answer any 2 questions from among the questions 21 to 24. These questions carry 6 marks each.

21. Show that

- $\cos z = \cos x \cosh y - i \sin x \sinh y$ and $\sin z = \sin x \cosh y + i \cos x \sinh y$
- $|\cos z|^2 = \cos^2 x + \sinh^2 y$ and $|\sin z|^2 = \sin^2 x + \sinh^2 y$.
- $\cos z$ and $\sin z$ are periodic with period 2π .

22. a) Integrate $\frac{\operatorname{Ln}(z+3) + \cos z}{(z+1)^2}$ counter clockwise around the circle $|z| = 2$.

b) State and prove Liouville's theorem.

23. Develop $\cos \pi z$ in a Taylor series with $\frac{1}{2}$ as center. Find the radius of convergence.

24. Evaluate the integral $\oint_C \left(\frac{ze^{z^2}}{z^4 - 16} + ze^{z/z} \right) dz$ where C is the ellipse $9x^2 + y^2 = 9$, counter clockwise.