M 8161

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2015

CORE COURSE IN MATHEMATICS

6B11 MAT : Complex Analysis

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks (Weightage 1):

   a) If $z = x + iy$, then $|z| =$ ____________

   b) If $f(z) = u(r, \theta) + iv(r, \theta)$, then the polar form of the Cauchy-Riemann equations are ____________

   c) The simularities of the functions $f(z) = (z + 1) / z^3 (z^2 + 1)$ are ____________

   d) If $f(z) = \sum_{n=-\infty}^{\infty} a_n (z - a)^n$, then residue of $f(z)$ at $z = a$ is ____________ **(Wt. 1)**

Answer **any 6** questions from the following **9** questions. (Weightage **one each**) :

2. If $z = x + iy$ is any non-zero complex number, obtain z^{-1} and verify that $zz^{-1} = 1$.

3. Examine whether $f(z) = z^2$ satisfy the Cauchy-Riemann equations.

4. If $f'(z) = 0$ everywhere in a domain D, show that $f(z)$ is a constant throughout D.

5. Show that $u(x, y) = \sinh x \sinh y$ is harmonic.

6. Find the values of z such that $e^z = -1$.

7. Evaluate $\int_C \left(e^{2z}/(z+1)\right) dz$, where C is the circle $|z| = 2$.

8. Prove that $\sin^{-1} z = -i \log\left[iz + (1-z^2)^{1/2}\right]$.

9. State the Laurent's theorem.

10. Find the residue of $f(z) = z/(z-1)(z+1)^2$ at the poles. **(6x1=6)**

P.T.O.

Answer **any seven** questions from the following **10** questions (weightage **2 each**).

11. If z_1 and z_2 are any two complex numbers, show that $|z_1 + z_2| \leq |z_1| + |z_2|$.

12. If $f : \mathbb{C} \to \mathbb{C}$ is continuous at a point $z_0 \in \mathbb{C}$ and $f(z_0) \neq 0$, show that $f(z) \neq 0$ throughout some neighbourhood of z_0.

13. Prove that $\frac{d}{dt}\left(e^{z_0 t}\right) = z_0 e^{z_0 t}$, where $z_0 = x_0 + iy_0$.

14. Using De-Moivre's theorem, express $\cos 3\theta$ in powers of $\cos \theta$.

15. Find $\int_C \bar{z}\, dz$, where C is the right hand half of the circle $|z| = 2$.

16. With the aid of remainders, show that $\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$, where $|z| < 1$.

17. If a function f(z) is analysis inside and on a positively oriented circle C with centre at z_0 and radius R, show that $|f^{(n)}(z_0)| \leq \frac{n!M}{R^n}, n = 1, 2, \ldots,$ where M is a positive real number such that $|f(z)| \leq M$.

18. If the function f(z) has a pole of order m at z_0, show that $f(z) = \varphi(z)/(z - z_0)^m$, where $\varphi(z)$ is analytic and non-zero at z_0.

19. Show that $z = \pi i/2$ is a simple pole of $f(z) = \frac{\tanh z}{z^2}$ and find the residue of f(z) at $z = \pi i/2$.

20. If two functions p and q are analytic at a point z_0, $p(z_0) \neq 0$, $q(z_0) = 0$ and $q'(z_0) \neq 0$, show that z_0 is a simple pole of the quotient $p(z)/q(z)$ and also prove that $\operatorname{Res}_{z=z_0} \frac{p(z)}{q(z)} = \frac{p(z_0)}{q'(z_0)}$. **(7×2=14)**

Answer **any three** questions from the following **five** questions (weightage **3 each**).

21. Prove that the square roots of $\sqrt{3} + i$ are $\pm \frac{1}{\sqrt{2}}\left(\sqrt{2+\sqrt{3}} + i\sqrt{2-\sqrt{3}}\right)$.

22. If $f(z) = u(x, y) + iv(x, y)$ and $f'(z)$ exists at a point $z_0 = x_0 + iy_0$, then show that u and v satisfy the Cauchy-Riemann equations $u_x = v_y$ and $u_y = v_x$ at (x_0, y_0).

23. If f(z) is analytic everywhere inside and on a simple closed curve C taken is positive sense, prove that $f'(z) = \frac{1}{2\pi i}\int_C \frac{f(s)}{(s-z)^2}\, ds$, where z is interior to C.

24. State and prove Cauchy's integral formula.

25. If $z_n = x_n + iy_n$ (n = 1, 2, 3, ...) and $z = x + iy$, then show that $\lim_{n\to\infty} z_n = z$ if and only if $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$. **(3×3=9)**

____