



M 6052



Reg. No. :

Name :

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.)
Examination, May 2014
CORE COURSE IN MATHEMATICS
6B11 MAT : Complex Analysis

Time : 3 Hours

Max. Weightage : 30

Instruction : Answer to **all** questions.

1. Fill in the blanks :

a) The principal argument of $\text{Arg}(z)$ when $z = \frac{i}{-2-2i} = \underline{\hspace{2cm}}$

b) When z_2 and z_3 are non-zero complex numbers then

$\left(\frac{z_1}{z_2 z_3} \right) = \underline{\hspace{2cm}}$

c) $\left| \frac{z_1}{z_2 z_3} \right| = \underline{\hspace{2cm}}$

d) $\overline{\overline{z} + 3i} = \underline{\hspace{2cm}}$

(W = 1)

From questions 2 to 10 ; answer **any six**.

2. Prove that z is real if and only if $\overline{z} = z$.
3. Prove that $\overline{z_1 + z_2 + \dots + z_n} = \overline{z_1} + \overline{z_2} + \overline{z_3} + \dots + \overline{z_n}$ for $n = 2, 3, 4, \dots$
4. Find the exponential form of the complex number $-1-i$.
5. Find the derivative of $f(x) = e^x (\cos y + i \sin y)$.
6. Define an entire function. Give an example.

P.T.O.



7. State Cauchy-Goursat theorem.
8. Prove that $f(z) = |z|^2$ is differentiable only at the origin.
9. If R is the radius of convergence of $\sum a_n z^n$, what is the radius of convergence $\sum a_n^2 z^n$?
10. Find the residue of $f(z) = \tan z$ at $z = \pi/4$. (W = 6x1=6)

From questions 11 to 20; answer any 7 :

11. Verify Cauchy-Riemann equations for the function $f(z) = z^2$.
12. Show that $U = e^x (x \cos y - y \sin y)$ satisfies the Laplace's equation.
13. Prove the fundamental theorem of algebra.
14. Prove that a bounded entire function is a constant.
15. Show that an analytic function $f(z)$ is a constant if its modulus is constant.
16. State Cauchy's Residue theorem.
17. Expand $\cos z$ about $z = \frac{\pi}{2}$ using Taylor's series.
18. What type of singularity have the $f(z) = \frac{1}{\sin z - \cos z}$ at $z = \pi/4$?
19. Find the residue of $f(z) = \frac{z^2}{z^2 + 4}$ at its poles.
20. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(ni)^2}{(2n)!} z^n$. (W = 7x2=14)



From questions 21 to 25; answer any 3 :

21. Find the harmonic conjugate of the function $u(x, y) = y^3 - 3x^2y$.
22. State and prove Cauchy's integral formula.
23. Find two Laurent series expansions, in powers of z for the function $f(z) = \frac{1}{z(1+z^2)}$.
24. When a singularity is said to be isolated? What are different kinds of isolated singularities. Give example for each.

25. Prove that $\int_0^{2\pi} \frac{d\theta}{1+a \cos \theta} = \frac{2\pi}{\sqrt{1-a^2}}$ ($-1 < a < 1$). (W = 3x3=9)