K21U 0220

Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Supple.)
Examination, April 2021
(2014-2015 Admissions)
BHM 604 : COMPLEX ANALYSIS – II

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark : **(10×1=10)**

1. Find the residue at $z = 1$ of $f(z) = \frac{4}{1-z}$.
2. Define pole of a function.
3. Define Bilinear Transformations.
4. Define conformal mapping.
5. State Jordan's Lemma.
6. State Rouche's theorem.
7. Find the zeros of $f(z) = \sin \pi z$.
8. Show that $f(z) = e^z$ is conformal at all points z.
9. Define Residue at a singular point.
10. What are harmonic functions ?

P.T.O.

SECTION – B

Answer **any 10** questions. **Each** question carries **3** marks : **(10×3=30)**

11. Show that $z = 0$ is an essential singularity of $f(z) = e^{\frac{1}{z}}$.

12. Obtain the bilinear transformation which maps 1, 0, –1 on to the points i, ∞, 1 respectively.

13. Find the residue of $f(z) = \dfrac{\sinh z}{z^4}$ at $z = 0$.

14. Find the residue at the singular points of $f(z) = \cot z$.

15. Evaluate the improper integral $\int_0^\infty \dfrac{dx}{x^4+1}$.

16. Define winding number and find the winding number of $w = \dfrac{1}{z^2}$ around $w = 0$.

17. Find the fixed points of the transformation $w = \dfrac{z-1}{z+1}$.

18. Using Rouche's theorem, determine the number of roots of $z^7 - 4z^3 + z - 1 = 0$ inside the circle $|z| = 1$.

19. Let two functions p and q be analytic at a point z_0. If $p(z_0) \neq 0$, $q(z_0) = 0$ and $q'(z_0) \neq 0$, then show that z_0 is a simple pole of $\dfrac{p(z)}{q(z)}$ and $\operatorname{Res}_{z=z_0} \dfrac{p(z)}{q(z)} = \dfrac{p(z)}{q'(z)}$.

20. Find and sketch the region on to which the half plane $y > 0$ is mapped by the transformation $w = (1 + i)z$.

21. Obtain the harmonic conjugate of $(x, y) = xy$.

22. Define Isogonal mapping and give one example.

23. Find the angle of rotation and scale factor of $w = z^2$ at the point $1 + i$.

24. Find the bilinear transformation which maps ∞, i, 0 to the points 0, i, ∞ respectively.

25. State why the transformation $w = iz$ is a rotation in the z plane through the angle $\dfrac{\pi}{2}$. Also find the image of the infinite strip $0 < x < 1$ under the transformation.

SECTION – C

Answer **any 6** questions. **Each** question carries **5** marks : **(6×5=30)**

26. Describe three types of Isolated singularities.

27. Show that $\int_0^{2\pi} \dfrac{d\theta}{1 + a\sin\theta} = \dfrac{2\pi}{\sqrt{1-a^2}}$, $-1 < a < 1$.

28. Evaluate the integral $\int_C \dfrac{dz}{z(z-1)^4}$ where C is the circle $|z - 2| = 1$ oriented in the counter clockwise direction.

29. Evaluate $\int_0^\pi \dfrac{\cos 2\theta}{5 + 4\cos\theta}\, d\theta$.

30. Evaluate $\int_C \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}\, dz$ around $|z| = 3$.

31. Evaluate $\int_C \dfrac{z\cosh \pi z}{z^4 + 13z^2 + 36}\, dz$ where C is any closed path enclosing all singularities and is oriented in the positive direction.

32. Show that the mapping $w = \dfrac{1}{z}$ maps circles and lines to circles and lines.

33. What do you mean by local inverse of a conformal mapping at a point ?

SECTION – D

Answer **any one** question. It carries **10** marks : **(1×10=10)**

34. State and derive Cauchy's Residue theorem.

35. Using contour integration, prove that $\int_0^\infty \dfrac{\sin x}{x}\, dx = \dfrac{\pi}{2}$.

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