



Reg. No. :

Name :



K20U 0129

VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.)

Examination, April 2020

(2014 Admission Onwards)

CORE COURSE IN MATHEMATICS

6B12MAT : Complex Analysis

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the first 4 questions are compulsory. They carry 1 mark each.

1. Sketch the region $\{z : \operatorname{Re}(iz) \geq 0\}$.
2. Define Harmonic function.
3. Find the Radius of convergence of $\sum 7^n z^n$.
4. Find the residue of $f(z) = e^z$ at $z = 0$.

SECTION – B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Give an example of a function which is differentiable exactly at one point and give its justification.
6. Verify Cauchy-Riemann equations for the function $f(z) = z^2$.
7. Evaluate $\int_C |z| dz$, where C is the line segment from origin to $1 + i$.
8. Find the Radius of convergence of $\sum (1 + i)^n (z - 3i)^n$.
9. Find the residue of $f(z) = \frac{9z + i}{z(z^2 + 1)}$ at $z = i$.

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10. Find the Laurent's series expansion of $f(z) = \frac{1}{z^5} \sin z$ with center 0.
11. State Taylor's Theorem. Find the Taylor's series expansion of $f(z) = \frac{1}{1+z^2}$ centered at $z = 0$.
12. Give an example of a series which is convergent but not absolutely. Give justification.
13. State Laplace's Equation. Give an example of a real valued function which satisfy Laplace's Equation on the complex plane.
14. State Cauchy's inequality.

SECTION - C

Answer **any 4** questions from among the questions 15 to 20. These questions carry **4 marks each**.

15. Prove that an analytic function of constant absolute value is constant in a domain.
16. Evaluate the following :
- a) $\int_0^{1+i} z^2 dz$
- b) $\int_{8+\pi i}^{8-3\pi i} e^z dz$
17. The power series $\sum a_n z^n$ converge at $z = 1$ and diverge at $z = -1$. Find the radius of convergence of $\sum a_n z^n$.
18. State and prove Residue Theorem.
19. Find an analytic function $f(z) = u(x, y) + iv(x, y)$, where $u(x, y) = xy$.
20. State and prove the theorem of convergence of power series.



SECTION - D

Answer **any 2** questions from among the questions 21 to 24. These questions carry **6 marks each**.

21. State and prove Cauchy - Riemann equations.
22. a) Define singular point, isolated singular point, removable singular point, pole and essential singular point.
b) Give an example of a non-isolated singular point.
23. a) State and prove Cauchy's integral formula.
b) Evaluate $\int_C \frac{e^z}{z-2} dz$, where C is the circle $|z| = 3$.
24. Give examples and justifications of power series having Radius of convergence 1 and
a) Which diverge at every point on the circle of convergence ?
b) Which doesn't diverge at every point on the circle of convergence ?