Reg. No. : ....

Name : ....

K19U 0124

VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.) Examination, April 2019
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B12 MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Write the polar form of the complex number $z = 1 + i$, using principle value of the argument.

2. Write the triangle inequality of complex numbers.

3. Find the Radius of convergence of $\sum n z^n$.

4. Give an example of a function having a simple pole at origin.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Verify Cauchy-Riemann equations for the function $f(z) = z^3$.

6. Does there exist a function in the complex plane which is analytic exactly at one point ? Give justification.

7. Evaluate $\int_C e^z \, dz$, where C is the line segment from origin to $1 + i$.

8. Evaluate $\int_C \frac{1}{z-i} dz$, using Cauchy's integral formula, where C is the circle $|z| = 2$.

9. Find the radius of convergence of $\sum \frac{(2n)!}{(n!)^2}(z-3i)^n$.

10. Find the Laurent's series expansion of $f(z) = \frac{1}{z^3 - z^4}$ about $z = 0$ in the region $0 < |z| < 1$.

11. Find the residue of $f(z) = \cot z$ at $z = 0$.

12. State Taylors Theorem. Find the Taylors series expansion of $f(z) = e^z$ centered at $z = 0$.

13. Define Essential singularity. Give one example of a function having essential singularity at $z = 0$.

14. Give an example of a series which is convergent but not absolutely. Give justification.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Prove that an analytic function whose modulus constant is constant in a domain.

16. State Cauchy's Integral Formula . Using this evaluate $\int_C \frac{z^3 - 6}{2z - i} dz$, where $C : = |z| = 1$.

17. State and prove Morera's Theorem.

18. State Cauchy-Hadamard formula for Radius of convergence. Using this Evaluate the radius of convergence of $\sum \left(\frac{a}{b}\right)^n (z-3i)^n$.

19. a) State Laurent's Theorem.

    b) Find the Residue of $f(z) = z^2 e^{\frac{1}{z}}$ with center 0.

20. a) State comparison test for convergence of a series.

    b) Discuss the convergence of the series $\sum \frac{\sin n}{3^n} z^n$.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. a) Define Analytic function.

    b) Give an example of a function which satisfy Cauchy-Riemann equation at origin but not analytic at origin and justification.

22. State and prove Cauchy's Integral formula.

23. Give examples and justifications of power serieses having Radius of convergence 1 and

    a) which diverge at every point on the circle of convergence

    b) which doesn't diverge at every point on the circle of convergence .

24. State and prove Residue theorem.

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