K18U 0206

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS-Supple./Improv.) Examination, May 2018
CORE COURSE IN MATHEMATICS
6B10 MAT : Analysis and Topology
(2012-13 Admns.)

Time : 3 Hours | Max. Weightage : 30

1. Fill in the blanks (Wt. 1) :

   a) $\int_0^1 t\sqrt{1+t^2}\,dt =$ ________

   b) Let f be Riemann integrable on [a, b] and ϕ be a differentiable function on (a, b) such that $\phi'(x) = f(x)$ for all $x \in [a, b]$. Then $\int_a^b f(x)\,dx =$ ________

   c) Let f be defined on (a, b] as follows
   $f(x) = 0,\ x \notin \{c_1, c_2, ..., c_n\}$
   $= 1\ x \in \{c_1, c_2, ..., c_n\}$
   where $c_1, c_2, ..., c_n \in [a, b]$
   $\int_a^b f =$ ________

   d) If $f(x) = x^2$ on [0, 1] and $P = \left\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\right\}$ is a partition of [0, 1] then $u(p, f) =$ ________ **(0.25×4=1)**

Answer **any 6** from the following **9** questions (Wt. 1).

2. Let f, g be bounded functions defined on [a, b] and let p be any partition of [a, b]. Show that $L(p, f+g) \geq L(p, f) + L(p, g)$.

3. If $f, g : I \to R$ are integrable on $I = [a, b]$ and if $f(x) \leq g(x)$ for all $x \in I$ then show that $\int_a^b f \leq \int_a^b g$.

4. If f(x) = [x] evaluate $\int_0^4 f(x)\,dx$.

5. Find the Taylor series expansion of f(x) = sinx, $x \in \mathbb{R}$.

6. Let X be a metric space. Show that any union of open sets in X is open.

7. Let X be a metric space with metric d given by

$$d(x,y) = 1 \text{ if } x \neq y$$
$$= 0 \text{ if } x = y$$

Identify the class of all open sets in this metric space.

8. Give an example of an infinite class of closed sets in $\mathbb{R}$ whose union is not closed.

9. Consider $Y = [0, 1] \cup (2, 3)$ as a subspace of $\mathbb{R}$ with usual topology. Show that [0, 1] is both open and closed in Y.

10. Let X be a topological space. Show that any closed subset of X, is the disjoint union of its interior and its boundary. **(6×1=6)**

Answer **any 7** from the following **10** questions (Wt. **2**).

11. If f is a non negative continuous function on [a, b] such that $\int_a^b f(x)\,dx = 0$ then show that $\int_a^b f(x)\,dx = 0 \ \forall x \in [a, b]$.

12. Check whether the following statement is true "Composition of integrable functions is integrable". Justify your claim.

13. Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose (f_n) converges uniformly on A to a function $f : A \to \mathbb{R}$. Show that f is continuous on A.

14. Let $f_n(x) = \frac{nx}{1+n^2x^2}$, $x \in (0, 1]$ show that $\lim \int_0^1 f_n(x)\,dx = \int_0^1 \lim f_n(x)\,dx$.

15. State and prove Cauchy Hadamard theorem.

16. Let X be a metric space and $G \subset X$. Prove the following "G is open iff G is a union of open spheres".

17. Given a metric space X, show that any finite subset of X is closed.

18. Show that a closed set in a topological space is nowhere dense iff its complement is every wheredense.

19. Let X be a nonempty set and consider the class of subsets of X consisting of empty set ϕ and all the sets whose complements are finite. Is this a topology ? Justify.

20. Let Y be a subspace of X. Then a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y. **(7×2=14)**

Answer **any 3** from the following **5** questions (Wt. **3**).

21. Show that the sequence (f_n) where $f_n(x) = nxe^{-nx^2}$ is not uniformly convergent on [0, 1].

22. Let I = [a, b] and $f : I \to \mathbb{R}$ be a bounded function on I. Show that f is integrable on I if and only if for each $\varepsilon > 0$ there is a partition P_ε of I such that $U(P_\varepsilon, f) - L(P_\varepsilon, f) < \varepsilon$.

23. State and prove Cantor's intersection theorem.

24. If $(A_n : n \in \mathbb{N})$ is a sequence of open dense subsets of a complete metric space V, show that $\bigcap A_n \neq \phi$.

25. Let $f : X \to Y$ be a mapping of one topological space into another show that the following statements are equivalent

i) f is continuous

ii) whenever F is closed in Y $f^{-1}(F)$ is closed in X

iii) $f(\overline{A}) \subseteq \overline{f(A)}$ for every subset A of x. **(3×3=9)**