Reg. No. : ....

Name : ....

K16U 0201

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improve.) Examination, May 2016

Core Course in Mathematics

6B10 MAT : ANALYSIS AND TOPOLOGY

Time : 3 Hours Max. Weightage. : 30

1. Fill in the blanks :

 a) The radius of convergence of the power series $\sum \frac{x^n}{n}$ is ____________

 b) Let $A \subseteq R$ and $\phi : A \to R$ is bounded on A. Then the uniform norm of ϕ on A is $\|\phi\|_A =$ ____________

 c) Let F, G be differentiable on [a, b] and let $f = F'$ and $g = G'$ belongs to R[a, b]. Then $\int_a^b fG =$ ____________

 d) Let X be an arbitrary metric space and $A \subseteq X$. Then Int (A) = ____________

(Weightage 1)

Answer **any six** from the following. **(Weightage 1 each)**

2. Prove the every constant function on [a, b] is R [a, b].

3. Evaluate $\int_1^4 \frac{\sin\sqrt{t}}{\sqrt{t}}\, dt$.

4. Let $G(x) = x^n (1 - x)$ for $x \in A = [0, 1]$. Prove that the convergence of {G(x)} to 0 is uniform on A.

P.T.O.

5. Define uniform convergence of a series of functions $\sum f_n$.

6. Define closed sphere in a metric space X. Give an example.

7. Give an example of two subsets A and B of the real line such that

$(A \cup B) \neq \text{Int}(A) \cup \text{Int}(B)$.

8. Let (X, d) be a metric space and $A \subseteq X$. Define the closure of A. Prove that A is closed if and only if $A = \overline{A}$.

9. Let T_1 and T_2 be two topologies in a non-empty set X and show that $T_1 \cap T_2$ is also a topology on X.

10. Let X be a topological space and let $X \subseteq X$. Define the boundary of A and prove that it is a closed set.

(Weightage 6×1=6)

Answer **any seven** from the following. **(Weightage 2 each)**

11. If $f \in R[a, b]$, then prove that f is bounded on [a, b].

12. If $f : [a, b] \to R$ is monotone on [a, b] then prove that $f \in R[a, b]$.

13. Prove that a sequences (f_n) of bounded functions on $A \subseteq R$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$.

14. Let R be the radius of convergence of $\sum a_n x^n$ and K be a closed and bounded interval contained in the interval convergence $(-R, R)$. Then prove that the power series converges uniformly on K.

15. State and prove Dini's theorem.

16. Prove that

(a) $\lim \left(\frac{x^2 + nx}{n}\right) = x$ for $x \in R$.

(b) $\lim \left(\frac{\sin(nx + n)}{n}\right) = 0$ for $x \in R$.

17. Let X be a topological space and A an arbitrary subset of X. Then prove that

$\overline{A} = \{x :$ each neighbourhood of x intersects $A\}$.

18. Let X be a topological space and A a subset of X. Then prove that (i) $\overline{A} = A \cup D(A)$ (ii) A is closed if and only if $A \supseteq D(A)$.

19. Prove that a closed subspace of complete metric space is complete.

20. Let X be a metric space. Then prove that any intersection of closed sets in X is closed.

(Weightage 7×2=14)

Answer **any three** from the following. **(Weightage 3 each)**

21. Let X be a complete metric space and let $\{F_n\}$ be a decreasing sequence of non-empty closed subsets of X such that $F = \cap_{n=1}^{\infty} F_n$ contains exactly one point.

22. State and prove Kuratowski's closure axioms on a topological space X.

23. Prove that a function $f : [a, b] \to R$ belongs to R[a, b] if and only if for every $\epsilon > 0$, there exists $y_\epsilon > 0$ such that if P and Q are any tagged partitions of [a, b] with $\|P\| < y_\epsilon$ and $\|Q\| < y_\epsilon$, then $|S(f,P) - S(f,Q)| < \epsilon$.

24. Let (f_n) be a sequence of functions in R[a, b] and suppose that $\{f_n\}$ converges uniformly on [a, b] to f. Then prove that $f \in R[a, b]$.

25. State and prove Fundamental Theorem of Calculus (Second form). **(Weightage 3×3=9)**
