M 6051

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2014

CORE COURSE IN MATHEMATICS

6B10 MAT : Analysis and Topology

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) If $P = \{1, 1.5, 2.1, 2.6, 3\}$ is a partition of $[1, 3]$, then $\|P\| =$ ________

   b) The radius of convergence of the series $\sum n!\, x^n$ is ________

   c) A subset of a topological space is said to be dense if ________

   d) A topological space is said to be separable if it has ________ **(Wt. 1)**

Answer **any six** from the following. Weight 1 **each**.

2. Prove that every constant function is Riemann integrable.

3. State the Cauchy criterion for the Riemann integrability of a real valued function.

4. Show that $\lim \left(\frac{nx}{1+nx} \right) = 1$ for every $x \in \mathbb{R}$, $x > 0$.

5. If (f_n) is a sequence defined on a subset D of real numbers with values in $\mathbb{R}$, define the convergence of $\sum f_n$.

6. Prove that $\overline{A}$ equals the intersection of all closed supersets of A.

7. Prove that in any metric space each closed sphere is a closed set.

8. If X is a metric space and $A \subset X$, prove that A is closed in X if and only if $A = \overline{A}$.

9. Prove that in a metric space, the complement of a closed set is open.

10. If X is a topological space and A and B are non-empty subsets of X, prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$. **(6×1=6)**

Answer **any seven** from the following. Weight **2 each**.

11. If f is Riemann integrable on [a, b], prove that f is bounded on [a, b].

12. If $f \in R[a, b]$, prove that $|f| \in R[a, b]$.

13. If (f_n) is a sequence of continuous function on a set $A \subseteq \mathbb{R}$ and if (f_n) converges uniformly on A to a function $f : A \to \mathbb{R}$, prove that f is continuous on A.

14. Prove that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$.

15. If R is the radius of convergence of the power series $\Sigma a_n x^n$, then prove that the series to absolutely convergent if $|x| < R$ and is divergent if $|x| > R$.

16. If X is a metric space, prove that a subset G of X is open if and only if it is a union of open sets.

17. Show that if $\{A_n\}$ is a sequence of nowhere dense sets is a complete metric space X, there exists a point in X which is not in any of the A_n's.

18. If a convergent sequence in a metric space has infinitely many distinct points, then prove that its limit is a limit point of the set of points of that sequence.

19. If X is a topological space and A is an arbitrary subsets of X, then prove that $\overline{A} = \{x :$ each neighbourhood of x intersects $A\}$.

20. Show that a subset of a topological space is perfect if and only if it is closed and has no isolated points. **(7×2=14)**

Answer **any three** from the following. Weight **3 each**.

21. If there is a finite set E in [a, b] and functions $f, F : [a, b] \to \mathbb{R}$ such that :
    i) F is continuous on [a, b]
    ii) $F'(x) = f(x)$ for all $x \in [a, b] \setminus E$ and
    iii) $f \in R[a, b]$, then prove that $\int_a^b f = F(b) - F(a)$.

22. If (f_n) is a sequence of function in R[a, b] and suppose (f_n) converges uniformly on [a, b] to f, then prove that $f \in R[a, b]$.

23. If X is a metric space with metric d, prove that $d_1(x, y) = d(x, y) / 1 + d(x, y)$ is also a metric on X.

24. If X is a complete metric space and Y is a subspace of X, prove that Y is complete if and only if it is closed.

25. State and prove Cantor's intersection on theorem. **(3×3=9)**