Reg. No. : ....

Name : ....

M 7149

V Semester B.Sc. Degree (CCSS – Reg./Supple./Imp.) Examination, November 2014
CORE COURSE IN MATHEMATICS
5B06 MAT : Real Analysis

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) The set of all $x \in \mathbb{R}$ that satisfy $|4x-5| \leq 3$ is __________

   b) The ε-neighbourhood of $a \in \mathbb{R}$ is __________

   c) $\text{Sup}\left\{1-\frac{(-1)^n}{n} : n \in \mathbb{N}\right\} =$ __________

   d) Every nonempty subset of $\mathbb{R}$ that has ________ has a supremum in $\mathbb{R}$. **(Wt. 1)**

Answer **any six** questions from the following (weightage **one each**) :

2. If a is a real number such that $0 \leq a < \varepsilon$ for $\varepsilon > 0$, then show that $a = 0$.

3. State and prove the triangle inequality.

4. Prove that a sequence in $\mathbb{R}$ can have at most one limit.

5. Using the definition of limit of a sequences prove that $\lim\left(\frac{3n+2}{n+1}\right) = 3$.

6. If $X = (x_n)$, $Y = (y_n)$ and $Z = (z_n)$ are sequences of real numbers such that $x_n \leq y_n \leq z_n$ for all $n \in \mathbb{N}$ and if $\lim (x_n) = \lim (z_n)$, show that $Y = (y_n)$ is convergent and $\lim(x_n) = \lim(y_n) = \lim(z_n)$.

P.T.O.

7. Prove that any convergent sequence of real numbers is a Cauchy sequence.

8. Prove that any absolutely convergent series in $\mathbb{R}$ is convergent.

9. If I is an interval, $f : I \to \mathbb{R}$ is continuous on I and if $f(a) < k < f(b)$, where $a, b \in I$, $k \in \mathbb{R}$, then show that there exists a point $c \in I$ between 'a' and 'b' such that $f(c) = k$.

10. If $f : A \to \mathbb{R}$, where $A \subseteq \mathbb{R}$, is a Lipschitz function, prove that f is uniformly continuous. **(6×1=6)**

Answer **any seven** questions from the following (weightage **2 each**) :

11. If $x \in \mathbb{R}$, show that there exists some $n_x \in \mathbb{N}$ such that $x < n_x$.

12. Prove that the set $\{ x \in \mathbb{R} : 0 \leq x \leq 1 \}$ is not countable.

13. If $X = (x_n : n \in \mathbb{N})$ is a sequence of real numbers and $m \in \mathbb{N}$, prove that the m-tail $X_m = (x_{m+n} : n \in \mathbb{N})$ converges if and only if X converges.

14. Prove that a convergent sequence is bounded.

15. Prove that the sequence $((-1)^n)$ is divergent.

16. Show that the series $\sum_{n=1}^{\infty} 1/n^p$ is convergent when $p > 1$.

17. If $X = (x_n)$ is a decreasing sequence with $\lim(x_n) = 0$ and if the partial sums (S_n) of Σy_n are bounded, prove that the series $\Sigma x_n y_n$ converges.

18. If $I = [a, b]$ is a closed bounded interval and if $f : I \to \mathbb{R}$ is continuous on I, prove that f is bounded on I.

19. If $f : I \to \mathbb{R}$ is continuous on I, where I is an interval, show that f(I) is an interval.

20. If $f : I \to \mathbb{R}$ is increasing on I, where $I \leq \mathbb{R}$ is an interval, prove that

$$\lim_{x \to c^-} f = \sup \{f(x) : x \in I, x < c\},$$

where $c \in I$ is not an end point of I. **(7×2=14)**

Answer **any three** questions from the following (Weightage **3 each**) :

21. Show that there exists a positive real number x such that $x^2 = 2$.

22. If S is a subset of $\mathbb{R}$ that contains at least two points and has the property that $[x, y] \subseteq S$ whenever $x, y \in S$ with $x < y$, prove that S is an interval.

23. Prove that a monotone sequence of real numbers is convergent if and only if it is bounded.

24. If I= [a, b] is a closed bounded interval that if $f : I \to \mathbb{R}$ is continuous on I, prove that f has an absolute maximum and an absolute minimum on I.

25. State and prove the continuous inverse theorem. **(3×3=9)**
