M 9812

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CCSS – Reg./Supple./Imp.) Examination, November 2015

CORE COURSE IN MATHEMATICS

5B06 MAT : Real Analysis

Time : 3 Hours | Max. Weightage : 30

1. Fill in the blanks.

   a) The set of all $x \in \mathbb{R}$ that satisfy $|x^2 - 1| \leq 3$ is ____________

   b) If $x \in V_\varepsilon(a)$ for $a \in \mathbb{R}$ and for every $\varepsilon > 0$, then $x =$ ________

   c) $\inf\left\{\frac{1}{n} : n \in \mathbb{N}\right\} =$ ____________

   d) Every non-empty subset of $\mathbb{R}$ that has _______ has an infimum in $\mathbb{R}$. **(W = 1)**

Answer **any six** questions from the following (Weightage **one each**).

2. If $y > 0$, show that there exist some $n_y \in \mathbb{N}$ such that $n_y - 1 \leq y \leq n_y$.

3. If $a, b \in \mathbb{R}$, prove that $||a| - |b|| \leq |a - b|$.

4. Prove that a sequence in $\mathbb{R}$ can have at most one limit.

5. If $X = (x_n)$ is a convergent sequence of real numbers and if $x_n \geq 0$ for all $n \in \mathbb{N}$, show that $x = \lim (x_n) \geq 0$.

6. Prove that $\lim\left(\frac{\sin n}{n}\right) = 0$.

7. Show that the sequence (Y_n) is a Cauchy sequence.

8. State the Ratio test and Raabe's test for the convergence of a series.

P.T.O.

9. If I is an interval, $f : I \to \mathbb{R}$ is continuous on I and if $f(a) < k < f(b)$, $a, b \in I$, $k \in \mathbb{R}$, show that there exists a point $c \in I$ between 'a' and 'b' such that $f(c) = k$.

10. If $f : A \to \mathbb{R}$, where $A \subseteq \mathbb{R}$, is uniformly continuous on A and if (x_n) is a Cauchy sequence in A, prove that $(f(x_n))$ is a Cauchy sequence in $\mathbb{R}$. **(6x1=6)**

Answer **any seven** questions from the following (Weightage **2 each**).

11. Show that there does not exist a rational number x such that $x^2 = 2$.

12. Prove that the set $\mathbb{R}$ of all real numbers is not countable.

13. If (x_n) is a sequence of real numbers, (a_n) is a sequence of positive real numbers with $\lim (a_n) = 0$ and if for some constant $c > 0$ and $m \in \mathbb{N}$, $|x_n - x| \leq C.a_n$, for $n \geq m$, where $x \in \mathbb{R}$, show that $\lim (x_n) = x$.

14. If $X = (x_n : n \in \mathbb{N})$ is a sequence of real numbers and $m \in \mathbb{N}$, show that the m-tail $X_m = (x_{m+n} : n \in \mathbb{N})$ of X converges if and only if X converges.

15. If a sequence $X = (x_n)$ of real numbers converges to a real number x, prove that any subsequence $X' = (x_{n_k})$ of X also converges to x.

16. Show that the series $\sum_{n=1}^{\infty} 1/n^2$ is convergent.

17. If $X = (x_n)$ is a convergent monotone sequence and if the series $\sum y_n$ is convergent, prove that series $\sum x_n y_n$ is convergent.

18. If $I = [a, b]$ is a closed bounded interval and if $f : I \to \mathbb{R}$ is continuous on I, prove that the set $f(I) = \{f(x) : x \in I\}$ is a closed, bounded interval.

19. If I is a closed bounded interval and if $f : I \to \mathbb{R}$ is continuous on I, prove that f is uniformly continuous on I.

20. If $f : I \to \mathbb{R}$ is increasing on I, where $I \subseteq \mathbb{R}$ is an interval, prove that

$\lim_{x \to c-} f = \sup \{f(x) : x \in I, x < c\}$, where $c \in I$ is not an end point of I. **(7x2=14)**

Answer **any three** questions from the following (Weightage **3 each**).

21. If S is a subset of $\mathbb{R}$ that contains atleast two points and has the property that $[x, y] \subseteq S$ whenever $x, y \in S$ with $x < y$, then prove that S is an interval.

22. If $I_n = [a_n, b_n]$, $n \in \mathbb{N}$ is a nested sequence of closed bounded intervals, prove that there exist a number $\zeta \in \mathbb{R}$ such that $\zeta \in I_n$ for all $n \in \mathbb{N}$.

23. Prove that a sequence of real numbers is convergent if and only if it is a Cauchy sequence.

24. If $I = [a, b]$ is a closed bounded interval and if $f : I \to \mathbb{R}$ is continuous on I, prove that f has an absolute maximum and an absolute minimum on I.

25. State and prove the continuous inverse theorem. **(3x3=9)**