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Show that the series 2 to converges when p > 1.

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Name : .....



# K16U 1715

Reg. No.: .....

Name : .....

V Semester B.Sc. Degree (CBCSS – 2014 Admn. – Regular)

Examination, November 2016

CORE COURSE IN MATHEMATICS

5B05 MAT : Real Analysis

Time: 3 Hours Max. Marks: 48

### SECTION - A

Answer all the questions. Each question carries one mark.

- 1. Find the infimum of  $S = \left\{ \frac{1}{2^m} \frac{1}{3^n} : m, n \in \mathbb{N} \right\}$ .
- 2. Give an example of a bounded sequence in  $\mathbb R$  that is not a Cauchy sequence.
- 3. If  $\sum a_n$  with  $a_n > 0$  is convergent, then is  $\sum \sqrt{a_n}$  always convergent? Either prove it or give a counter example.
- 4. Define  $g: \mathbb{R} \to \mathbb{R}$  by

$$g(x) = \begin{cases} 2x & \text{if } x \text{ is rational} \\ x+3 & \text{if } x \text{ is irrational} \end{cases}$$

Determine the points at which g is continuous.

 $(4 \times 1 = 4)$ 

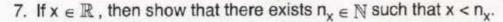
## SECTION-B

Answer any 8 questions. Each question carries two marks.

- 5. Show that there does not exist a rational number r such that  $r^2 = 2$ .
- 6. Show that the set  $A = \{ x \in \mathbb{R} : x^2 < 1 x \}$  is bounded above, and then find its least upper bound.







- 8. Show that  $\lim_{n \to \infty} (n^{1/n}) = 1$ .
- Suppose that every subsequence of X = (x<sub>n</sub>) has a subsequence that converges to 0. Show that lim X = 0.
- 10. If the series  $\sum x_k$  converges then show that  $\lim(x_k) = 0$ . Is the converse true? Justify.
- 11. Establish the convergence or divergence of the series whose  $n^{th}$  term is  $\frac{n}{(n+1)(n+2)}.$
- 12. Let  $\sum x_n$  be an absolutely convergent series in  $\mathbb{R}$ . Show that any rear-rangement  $\sum y_k$  of  $\sum x_n$  converges to the same value.
- 13. Give an example of functions f and g that are both discontinuous at a point c in  $\mathbb R$  but, both f + g and fg are continuous at c.
- 14. Let I be an interval and let  $f: I \to \mathbb{R}$  be continuous on I. If  $a, b \in I$  and if  $k \in \mathbb{R}$  satisfies f(a) < k < f(b), show that there exists a point  $c \in I$  between a and b such that f(c) = k. (8x2=16)

## SECTION-C

Answer any 4 questions. Each question carries four marks.

- 15. If  $a, b \in \mathbb{R}$ , prove the following:
  - i)  $|a+b| \le |a| + |b|$
  - ii)  $||a| |b|| \le |a b|$
- Let S and T be bounded nonempty subsets of R such that S ⊆ T. Prove that inf T ≤ inf S ≤ sup S ≤ sup T.



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- 17. State and prove the Squeeze theorem on limits of sequences. Apply it to find  $\text{lim}\bigg(\frac{\sin n}{n}\bigg).$
- 18. Show that the series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  converges when p > 1.
- Discuss the convergence or the divergence of the series with n<sup>th</sup> term (for sufficiently large n) given by (n /n n)<sup>-1</sup>.
- 20. Let I be a closed bounded interval and let  $f: I \to \mathbb{R}$  be continuous of I. Show that f is bounded on I. (4×4=16)

### SECTION-D

Answer any 2 questions. Each question carries six marks.

- State and prove the nested intervals property. Using the same show that the set of real numbers is uncountable.
- 22. a) Show that every sequence of real numbers has a monotone subsequence.
  - b) Let  $(x_n)$  be a Cauchy sequence such that  $x_n$  is an integer for every  $n \in \mathbb{N}$ . Show that  $(x_n)$  is ultimately constant.
- 23. a) State and prove the Dirichlet's test for convergence of a series.
  - b) Test for convergence the series  $1 \frac{1}{2} \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \frac{1}{6} \frac{1}{7} + \dots$ , where the signs come in pairs.
- 24. Let  $I \subseteq \mathbb{R}$  be an interval and let  $f: I \to \mathbb{R}$  be strictly monotone and continuous on I. Show that the function g inverse to f is strictly monotone and continuous on f(I). (2×6=12)