K16U 1715

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – 2014 Admn. – Regular) Examination, November 2016

CORE COURSE IN MATHEMATICS

5B05 MAT : Real Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **one** mark.

1. Find the infimum of $S = \left\{ \frac{1}{2^m} - \frac{1}{3^n} : m, n \in \mathbb{N} \right\}$.

2. Give an example of a bounded sequence in $\mathbb{R}$ that is not a Cauchy sequence.

3. If $\sum a_n$ with $a_n > 0$ is convergent, then is $\sum \sqrt{a_n}$ always convergent ? Either prove it or give a counter example.

4. Define $g : \mathbb{R} \to \mathbb{R}$ by

$$g(x) = \begin{cases} 2x & \text{if } x \text{ is rational} \\ x + 3 & \text{if } x \text{ is irrational} \end{cases}$$

Determine the points at which g is continuous. **(4×1=4)**

SECTION – B

Answer **any 8** questions. **Each** question carries **two** marks.

5. Show that there does not exist a rational number r such that $r^2 = 2$.

6. Show that the set $A = \{ x \in \mathbb{R} : x^2 < 1 - x \}$ is bounded above, and then find its least upper bound.

7. If $x \in \mathbb{R}$, then show that there exists $n_x \in \mathbb{N}$ such that $x < n_x$.

8. Show that $\lim(n^{1/n}) = 1$.

9. Suppose that every subsequence of $X = (x_n)$ has a subsequence that converges to 0. Show that $\lim X = 0$.

10. If the series $\sum x_k$ converges then show that $\lim(x_k) = 0$. Is the converse true ? Justify.

11. Establish the convergence or divergence of the series whose n^{th} term is $\frac{n}{(n+1)(n+2)}$.

12. Let $\sum x_n$ be an absolutely convergent series in $\mathbb{R}$. Show that any rear-rangement $\sum y_k$ of $\sum x_n$ converges to the same value.

13. Give an example of functions f and g that are both discontinuous at a point c in $\mathbb{R}$ but, both f + g and fg are continuous at c.

14. Let I be an interval and let $f : I \to \mathbb{R}$ be continuous on I. If $a, b \in I$ and if $k \in \mathbb{R}$ satisfies $f(a) < k < f(b)$, show that there exists a point $c \in I$ between a and b such that $f(c) = k$. **(8×2=16)**

SECTION – C

Answer **any 4** questions. **Each** question carries **four** marks.

15. If $a, b \in \mathbb{R}$, prove the following :

   i) $|a+b| \le |a| + |b|$

   ii) $||a| - |b|| \le |a - b|$.

16. Let S and T be bounded nonempty subsets of $\mathbb{R}$ such that $S \subseteq T$. Prove that $\inf T \le \inf S \le \sup S \le \sup T$.

17. State and prove the Squeeze theorem on limits of sequences. Apply it to find $\lim\left(\frac{\sin n}{n}\right)$.

18. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges when $p > 1$.

19. Discuss the convergence or the divergence of the series with n^{th} term (for sufficiently large n) given by $(n \ln n)^{-1}$.

20. Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous of I. Show that f is bounded on I. **(4×4=16)**

SECTION – D

Answer **any 2** questions. **Each** question carries **six** marks.

21. State and prove the nested intervals property. Using the same show that the set of real numbers is uncountable.

22. a) Show that every sequence of real numbers has a monotone subsequence.

   b) Let (x_n) be a Cauchy sequence such that x_n is an integer for every $n \in \mathbb{N}$. Show that (x_n) is ultimately constant.

23. a) State and prove the Dirichlet's test for convergence of a series.

   b) Test for convergence the series $1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{6} - \frac{1}{7} + \ldots$, where the signs come in pairs.

24. Let $I \subseteq \mathbb{R}$ be an interval and let $f : I \to \mathbb{R}$ be strictly monotone and continuous on I. Show that the function g inverse to f is strictly monotone and continuous on $f(I)$. **(2×6=12)**