

SECTION - D

(Answer any two questions. Each carries 8 marks)

27. (a) Prove the existence of a real number such that $x^2 = 2$.

(b) If $a < b$, show that $|a - b| < |a - b|$.

28. (a) State and prove Weierstrass Weierstrass Theorem for sequences.

(b) Let (x_n) be a bounded sequence in \mathbb{R} . Show that it contains

subsequence (x_{n_k}) such that $x_{n_k} \rightarrow L$.

29. (a) State and prove Bolzano-Weierstrass Theorem.

(b) Check the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

30. (a) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 - 1$. Is the function on \mathbb{R}

one-to-one? Justify your answer.

(b) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 - 1$.

(c) Calculate the first few terms of the sequence (x_n) defined by $x_1 = 1$ and $x_{n+1} = \frac{1}{x_n}$.



Reg. No. :

Name :

V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.)
Examination, November 2020
(2014 Admn. Onwards)
CORE COURSE IN MATHEMATICS
5B05MAT : Real Analysis

Time : 3 Hours

Max. Marks : 48

SECTION – A

(Answer all the questions. Each carries 1 mark)

1. Find all real x so that $|x - 1| < |x|$.
2. Give two divergent sequences (x_n) and (y_n) such that $(x_n + y_n)$ is convergent.
3. State n^{th} term test.
4. Show that $f(x) = \frac{1}{x}, \forall x$ is not uniformly continuous on $(0, \infty)$. (4×1=4)

SECTION – B

(Answer any eight questions. Each carries 2 marks)

5. There does not exist a rational number r such that $r^2 = 2$. Prove.
6. For positive real numbers a and b , show that $\sqrt{ab} \leq \frac{1}{2}(a + b)$, where equality occurring if and only if $a = b$.
7. Define infimum of a set. Find $\inf S$ if $S = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$.
8. A sequence in \mathbb{R} can have at most one limit. Prove.



9. Prove that every Cauchy sequence is bounded.
10. If $\sum x_n$ and $\sum y_n$ are convergent, show that the series $\sum(x_n + y_n)$ is convergent.
11. Check the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$.
12. If $X = (x_n)$ is a decreasing sequence of real numbers with $\lim x_n = 0$, and if the partial sums (s_n) of $\sum y_n$ are bounded, prove that the series $\sum x_n y_n$ is convergent.
13. Let I be a closed bounded interval and let $f: I \rightarrow \mathbb{R}$ be continuous on I . Show that the set $f(I) = \{f(x) : x \in I\}$ is a closed bounded interval.
14. Give an example to show that every uniformly continuous functions are not Lipschitz functions. (8×2=16)

SECTION - C

(Answer any four questions. Each carries 4 marks)

15. State and prove Archimedean property of \mathbb{R} .
16. If S is a subset of \mathbb{R} that contains at least two points and has the property
If $x, y \in S$ and $x < y$, then $[x, y] \subseteq S$.
Show that S is an interval.
17. For $C > 0$, show that $\lim(C^{\frac{1}{n}}) = 1$.
18. Discuss the convergence of the Geometric series $\sum_{n=0}^{\infty} r^n$ for $r \in \mathbb{R}$.
19. If $\sum x_n$ is an absolutely convergent series in \mathbb{R} , show that any rearrangement $\sum y_k$ of $\sum x_n$ converges to the same value.
20. Let I be a closed bounded interval and let $f: I \rightarrow \mathbb{R}$ be continuous on I . Show that f is uniformly continuous on I . (4×4=16)



SECTION - D

(Answer any two questions. Each carries 6 marks)

21. a) Prove the existence of a real number x such that $x^2 = 2$.
b) If $a, b \in \mathbb{R}$, show that $||a| - |b|| \leq |a - b|$.
22. a) State and prove Bolzano Weierstrass Theorem for sequences.
b) If $X = (x_n)$ is a bounded increasing sequence in \mathbb{R} , show that it converges and $\lim(x_n) = \sup\{x_n : n \in \mathbb{N}\}$.
23. a) State and prove D'Alembert ratio test.
b) Check the convergence of the series whose n^{th} term is $\frac{(n!)^2}{(2n)!}$.
24. a) Let I be a closed bounded interval and let $f: I \rightarrow \mathbb{R}$ be continuous on I . If $\varepsilon > 0$, then there exists step functions $s_\varepsilon: I \rightarrow \mathbb{R}$ such that $|f(x) - s_\varepsilon(x)| < \varepsilon, \forall x \in I$.
b) Let $f(x) = x, \forall x \in [0, 1]$. Calculate the first few Bernstein polynomials for f . (2×6=12)